Prince Sultan University

Department of Mathematical Sciences

MATH 223 – First Examination

1 April 2007

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Time allowed: 90 minutes **Maximum points: 100 points**

1. (6 points) Reduse $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$ to reduced row-echelon form without introducing any fractions

2. (8 points) For which value(s) of λ does the system of equations

$$\begin{aligned} (\lambda-3)x + y &= 0 \\ x + (\lambda-3)y &= 0 \end{aligned}$$

have nontrivial solutions?

3. (6 points) Using the given information $(I+2A)^{-1}=\begin{bmatrix}-1 & 2\\ 4 & 5\end{bmatrix}$, find the matrix A

4. (6 points) Find a diagonal matrix A that satisfies $A^{-2}=\begin{bmatrix}25 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 9\end{bmatrix}$

5. (8 points) Solve for x . $\begin{vmatrix}x & -1\\ 3 & 1-x\end{vmatrix}=\begin{vmatrix}1 & 0 & -3\\ 2 & x & -6\\ 1 & 3 & x-5\end{vmatrix}$

6. (8 points) Evaluate the determinant of the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$ by reducing the matrix to row-echelon form.

7. (6 points) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Assuming that $\det(A) = -6$, find

i) $\det(3A)$

ii) $\det(5A^{-1})$

iii) $\det((2A)^{-1})$

8. (6 points) Prove the identity without evaluating the determinants.

$$\begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix} = -2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

9. (6 points) Find the distance between the point $(1,8)$ and the line $3x + y = 5$.

10. (6 points) Let $\mathbf{p} = (2, k)$ and $\mathbf{q} = (3,5)$. Find k such that $\mathbf{p}$ and $\mathbf{q}$ are parallel.

11. (8 points) let $A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ find $adj(A)$ then use it to find A^{-1}.

12. (12 points) Solve by Cramer's rule, where it applies.

$$\begin{aligned} 4x + 5y &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

13. (6 points) Let $u = (3,1,-7)$ and $a = (1,0,5)$

i) Find the vector component of u along a.

ii) Find the vector component of u orthogonal to a.

iii) Find $\|proj_a\ u\|$

14. (8 points) Show that there do not exist scalars c_1, c_2, and c_3 such that:

$$c_1(-2,9,6) + c_2(-3,2,1) + c_3(1,7,5) = (0,5,4)$$