

1)a) Evaluate the iterated integral: (i) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \cos(x^2 + y^2) dy dx$. (ii) $\int_0^1 \int_{\sqrt{y}}^1 \frac{1}{x^3+1} dx dy$.

b) Find the area of the surface part of the sphere $x^2 + y^2 + z^2 = 1$ that lies inside within the cylinder $x^2 + y^2 = x$ and above the xy -plane. (sketch the cylinder).

2)a) Find a basis for the subspace $W = \{(x, y, z): 3x - 2y + 5z = 0\}$ of R^3 .

b) Find the distance between the vectors $u = (2, 1, -1)$ and the vector $v = (2, 1, 1) \times (-1, 2, 4)$.

3)a) Determine whether the following set $S = \{p_1 = 1 - x + 2x^2, p_2 = 3 + x, p_3 = 5 - x + 4x^2, p_4 = -2 - 2x + 2x^2\}$ spans the vector space P_2 of all polynomials with degree less than or 2. What is the dimension of the vector subspace $W = \text{span}(S)$?

b) For which real values of μ do the following vectors form a linearly dependent set in R^3 ?

$$u_1 = \left(\mu, -\frac{1}{2}, -\frac{1}{2}\right), u_2 = \left(-\frac{1}{2}, \mu, -\frac{1}{2}\right), u_3 = \left(-\frac{1}{2}, -\frac{1}{2}, \mu\right)$$

4) a) Use the Gram-Schmidt process to transform the basis $S = \{v_1 = (1, 1, 1), v_2 = (-1, 1, 0), v_3 = (1, 2, 1)\}$ of the 3-space into an orthonormal basis T .

b) If $w = (1, 2, 3)$ find the coordinate vector of w with respect to the basis S ($(w)_S = ?$).

c) If $w = (1, 2, 3)$ find the coordinate vector of w with respect to the orthonormal basis T ($(w)_T = ?$).

5) Consider the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$ with its characteristic polynomial $p(\mu) = -(\mu - 5)(\mu - 3)^2$.

a) For each eigenvalue μ of A , find the set of eigenvectors corresponding to μ .

b) If possible, find a basis for R^3 consisting of eigenvectors of A . Is A diagonalizable?

c) If successful in b), find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

d) Find the eigenvalues of A^2 and $(A - 3I)$ and their corresponding eigenvectors.