PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences
MATH 211 – Business Calculus
Final Examination
January 2011

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Time allowed: 2 hours

Q1. (a) **(5 points)** Study the limit of the function

$$f(x) = \begin{cases} 2x - 10 & if\ x \geq 4 \\ x^3 + 10 & if\ x < 4 \end{cases} \text{ at } x = 4.$$

(b) **(6 points)** Find the absolute maximum and minimum of the function $f(x) = 2x^3 - 3x^2 + 10$ over the interval $-3 \leq x \leq 4$.

(c) **(9 points)** Graph the function $f(x) = x^4 - 24x^2 + 4$

Q2. (a) **(9 points)** Find the derivative of the functions

i. $y = \frac{2x-9}{x^2-1}$

ii. $y = x^3\sqrt{4x-2}$

iii. $y = \ln(3x-1)^{2x}$

(b) **(5 points)** Find the equation of the tangent line to the curve

$$y = x^3 e^{2x} \text{ at } x = 2.$$

(c) **(8 points)** The demand function of a particular commodity is $q = D(p) = 5000e^{-0.04p}$ units, where p is the price \$ per unit for which all q units will be sold.

i. Study the elasticity of the demand when the price is 15 \$ per unit.

ii. Determine when the demand is of unit elasticity.

Q3. (a) **(7 points)** Records indicate that t months after the beginning of the year, the price of ground beef in Hyperpanda was $p(t) = 0.09t^2 - 0.2t + 40$ SR per Kg. What was the average price of ground beef during the first 3 months of the year?

(b) **(20 points)** Evaluate the following integrals

i) $\int_4^9 \frac{3}{\sqrt{x}} - 4\sqrt{x} + \frac{1}{2}\,dx$

ii) $\int \frac{\sqrt{\ln x}}{x}\,dx$

iii) $\int \frac{\left(x^2-1\right)^2}{x^3}\,dx$

iv) $\int \frac{x^3}{\left(x^2-1\right)^2} dx$

(c) **(8 points)**Find the area of the region bounded by the curves $y = x^2 - 4x - 12$ and the x-axis.

Q4. (a) **(7 points)**Given the function $f(x, y) = (x^2 - y^2)^4$

i. Find the first partial derivatives f_x, f_y.

ii. Show that $y.f_x + x.f_y = 0$

(b) **(9 points)**A grocer's daily profit from the sales of two brands of cat food is $P(x, y) = x(70 - 5x + 4y) - 70 + y(80 + 6x - 7y)$ dollars,
where *x* is the price per can of the first brand and y is the price of the second. Currently the first brand sells for \$0.5 per can and the second for \$0.52 per can.
Use marginal analysis to estimate the change in the daily profit that will result if the grocer raises the price of the second brand by 1 cent per can but keeps the price of the first brand unchanged.

(c) **(7 points)**Suppose that t years from now, one investment plan will be generating profit at the rate of $P'_1 = 120 + t^2$ hundred dollars per year, while a second investment will be generating profit at the rate of $P'_2 = 360 + 6t$ hundred dollars per year.
Compute the net excess profit of plan 2 over plan 1 for the period of the first 5 years.