

- 1) Consider the vectors  $u = (1, 2, -8)$ ,  $v = (-2, 3, 1)$  and  $w = (2, 0, 8)$ .
- a) Determine the cosine of the angle between  $u$  and  $v$  and decide whether it is acute, obtuse or they are orthogonal.
  - b) Find the vector component of  $u$  along  $w$  and the vector component of  $u$  orthogonal to  $w$ .
  - c) Find a vector orthogonal to both  $u$  and  $v$  and then find the area of the triangle determined by the vectors  $u$  and  $v$ .
  - d) Find the norm of  $u - 2v$  and the distance between  $u$  and  $v$ .
  - e) Find the volume of the parallelepiped in the 3-space determined by the vectors  $u$ ,  $v$  and  $w$ .

- 2) a) Find the equation of the plane passing through the point  $P(1, 2, -3)$  and parallel to the plane whose equation is  $-3x + 2y + 2z + 10 = 0$ .
- b) Show that the planes  $x - 2y + 3z - 2 = 0$  and  $2x - 4y + 6z - 1 = 0$  are parallel and find the distance between them.
- c) Find parametric equations for the line passing through  $P(-3, -1, 2)$  and which is parallel to the vector  $n = (2, -1, 3)$ .
- d) Find the equation of the plane passing through the points  $(1, -1, 1)$ ,  $(2, -1, 0)$  and  $(-1, 2, 4)$ .

- 3) Consider the linear operators  $T: R^2 \rightarrow R^2$  and  $S: R^2 \rightarrow R^2$  where  $T$  has the standard matrix representation  $[T] = \begin{bmatrix} 2 & 2 \\ 4 & -2 \end{bmatrix}$  and  $S$  is the rotational operator counterclockwise by an angle  $\frac{\pi}{4}$ .
- a) Is the operator  $T$  one to one ? Onto? Why?
  - b) Find the vector  $v = (ToS)(u)$ , where  $u = (0, \frac{\sqrt{2}}{2})$ .
  - c) Find  $T(-2,2)$  and  $S(-1,1)$ .
  - d) Find  $(SoT)^{-1}(w)$  if exists, where  $w = (-1, -1)$ .

4) a) Determine whether the polynomials

$p_1(x) = 1 + x + 2x^2$ ,  $p_2(x) = 1 + x^2$ ,  $p_3(x) = 2 + x + 3x^2$ , span the vector space  $P_2$  of all real polynomials with degree less than or equal to 2.

b) Find and describe the kernel of the linear operator  $T: R^3 \rightarrow R^3$  defined by

$$T(x, y, z) = (x - 2y + 3z, -3x + 6y + 9z, -2x + 4y - 6z).$$