PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences
MATH 211 – Business Calculus
Final Examination
June 2010

Maximum Time 150 Minutes

Q1. (a) Find the absolute maximum and minimum of the function

$f(x) = x^4 - 2x^2 + 2$ on the interval $-0.5 \le x \le 3$.

(b) The manager of a bookstore determines that when a certain novel is priced at p dollars per copy, the daily demand will be $q = 200 - p^2$ copies, where $0 \le p \le \sqrt{200}$. Determine where the demand is elastic, inelastic and of unit elasticity with respect to price.

(c) A projection made in January of 2000 determined that *x* years later, the average property tax on a two-bedroom apartment will be $T(x) = 60x^{3/2} + 40x + 1200$ dollars. Estimate the percentage change by which the property tax will increase in January 2011.

Q2. (a) Find the derivative $\frac{dy}{dx}$ of the functions (simplify your answer if possible)

i. $y = \frac{e^{2x}-1}{e^{-2x}+1}$

ii. $y = ln\left(\frac{x^2-1}{x^3\sqrt{x+1}}\right)$

iii. $xy^3 - 3xy + 5 = y^2$

(b) Find the equation of the tangent line to the curve

$f(x) = \sqrt{x^2 + 2x + 6}$ at $x = 2$.

(c) Use calculus to graph the function $y = x^3 - 6x^2 + 9x$

Q3. (a) Evaluate the following integrals

i. $\int x\left(x^2 - \sqrt{x}\right)dx$

ii. $\int_1^5 \frac{x^3-3x^2+5x}{x^2}dx$

iii. $\int \frac{4x}{\sqrt{x^2-5}}dx$

(b) A manufacturer supplies $S(p) = 0.5p^2 + 3p + 7$ hundred units of a certain commodity to the market when the price is p dollars per unit. Find the average supply as the price varies from $2 to $5.

(c) Suppose that when it is t years old, a particular industrial machine generates revenue at the rate $R'(t) = 6025 - 8t^2$ $ per year and that operating and servicing costs accumulate at the rate $C'(t) = 4681 + 13t^2$ $ per year.

i. Compute the machine useful life time.
ii. Compute the net profit generated by the machine over its useful life time.

Q4. a) Let $f(x, y) = \sqrt{x - y + 1}$

i. Find the domain of f(x) and graph it.
ii. Find the first partial derivatives, $f_x(4,4)$, $f_y(4,4)$.

b) At a certain factory, the daily output is $Q = 60K^{.4}L^{0.6}$ units, where K is the capital expenditure in units of $1000 and L is the size of the labor force, measures in worker-hours. Suppose the current capital expenditure is $45,000 and the level of labor is 341 worker-hours.

i. Find the marginal productivity of capital Q_K and the marginal productivity of labor Q_L.
ii. Estimate the change of the output that will result if capital expenditure is increased by $1200 and labor is increased by 1.3 worker-hours.
iii. Estimate the change of the output that will result if capital expenditure is increased by $1200 and labor remains unchanged.

Good Luck