Prince Sultan University
Orientation Mathematics Program
MATH 001

Midterm Examination
Semester II, Term 092
Saturday, April 10, 2010
Time Allowed: 120 minutes (2 hours)

Student Name: _____

Student ID #: _____ Key _____ Section #: _____

Teacher's Name: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 14 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3,4	14	
5,6,7,8	18	
9,10	18	
11	15	
12	20	
13,14	15	
Total	100	

Show all your steps for each question

Q.1 (2 points) Find $(\{1,3,5,7\}\cap\{2,3,6,7,10\})\cup\{3,9,12\}$

$\{3,7\}\cup\{3,9,12\} = \{3,7,9,12\}$

Q.2 (2 points) Let $A=\left\{-11,\ -\frac{3}{7},\ 0.45,\ \sqrt{23},\ \sqrt{25},\ \pi\right\}$

(i) List all the rational numbers in set A $\{-11, -\frac{3}{7}, 0.45, \sqrt{25}\}$

(ii) List all the integers in set A $\{-11, \sqrt{25}\}$

Q.3 (4 points) Evaluate the following:

(i) $\left(5+\sqrt[4]{81}\right)^{-\frac{2}{3}} = (5+3)^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{4}$

(ii) $\frac{|x-1|}{3}-\frac{xy}{1+y}$, for $x=-2$ and $y=1$

$\frac{|-2-1|}{3} - \frac{(-2)(1)}{1+1} = \frac{|-3|}{3} - \frac{-2}{2}$

$= \frac{3}{3} + 1$

$= 2$

Q.4 (6 points) Perform the indicated operations and write the result in the ***standard form*** of a complex number.

(i) $\sqrt{-9}\left(2-\sqrt{-4}\right) = 3i(2-2i)$

$= 6i - 6i^2$

$= 6i + 6 = 6+6i$

(ii) $(2-3i)(1-i)+(3-i)(3+i)$ $= (2 - 2i + 3i^2 - 3i) + (9 - i^2)$

$= (2 - 5i - 3) + (9 + 1)$

$= -1 - 5i + 10 = 9 - 5i$

(iii) $\frac{3-2i}{2+i} = \frac{3-2i}{2+i}\cdot\frac{2-i}{2-i} = \frac{6-3i-4i+2i^2}{4-i^2}$

$= \frac{6-7i-2}{4+1} = \frac{4-7i}{5}$

$= \frac{4}{5} - \frac{7}{5}i$

Q.5 (2 points) Use equality of two complex numbers to find the real numbers x and y such that the equation is true.

$x+\sqrt{-16}=2+yi$ $\quad\sqrt{-16}=4i$

$\boxed{x=2}$ $\quad\boxed{y=4}$

Q.6 (3 points) Rationalize the denominator in $\frac{4}{3\sqrt{2}-4}$

$$\frac{4}{3\sqrt{2}-4}\cdot\frac{3\sqrt{2}+4}{3\sqrt{2}+4}=\frac{12\sqrt{2}+16}{9(2)-16}=\boxed{\frac{12\sqrt{2}+16}{2}}=\boxed{6\sqrt{2}+8}$$

Q.7 (3 points) Find all numbers that must be excluded from the domain of the rational Expression $\frac{x-2}{x^2-2x-15}$

$\rightarrow (x-5)(x+3)$

$x\neq 5$, $x\neq -3$

The numbers that must be excluded from the domain are. $\boxed{-3 \text{ and } 5}$

Q.8 (10 points) Simplify each of the following expressions. Assume that all variables represent positive numbers.

(i) $8-5[3x-4(2x-3)]$ $= 8-5[3x-8x+12]$

$= 8-5[-5x+12]$

$= 8+25x-60 = \boxed{-52+25x}$

(ii) $(3x^2y^{-3})(-2x^3y)^2$ $= (3x^2y^{-3})(4x^6y^2)$

$= \boxed{\frac{12x^8}{y}}$

(iii) $\sqrt{2x^3}+3x\sqrt{8x}$ $= \sqrt{x^2\cdot 2x}+3x\sqrt{4\cdot 2x}$

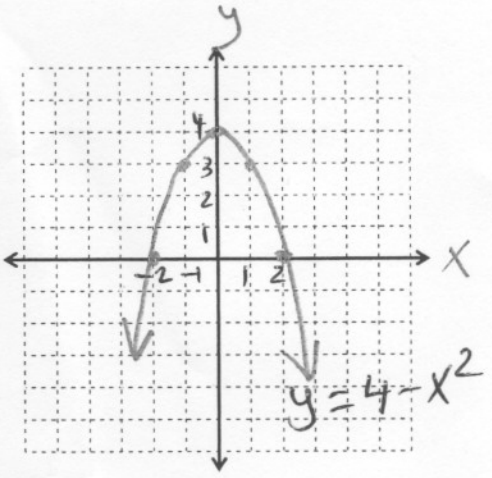
$= x\sqrt{2x}+6x\sqrt{2x}$

$= \boxed{7x\sqrt{2x}}$

(iv) $\left(\frac{-35x^2y^4}{5x^6y^{-8}}\right)^3$ $= \left(\frac{-7y^{12}}{x^4}\right)^3 = \boxed{\frac{-7^3y^{36}}{x^{12}}} = \boxed{\frac{-343y^{36}}{x^{12}}}$

Q.9 (6 points) Consider the equation $y = 4 - x^2$.

(i) Graph this equation. Select integers for x, starting with -2 and ending with 2.



x	y	(x, y)
-2	0	(-2, 0)
-1	3	(-1, 3)
0	4	(0, 4)
1	3	(1, 3)
2	0	(2, 0)

(ii) Determine the $x-$intercepts, if any.

$x = 2$ and -2
$(2, 0)$ $(-2, 0)$

(iii) Determine the $y-$intercept, if any.

$y = 4$
$(0, 4)$

Q.10 (12 points) Factor each of the following ***completely:***

(i) $5x^4 - 45x^2$ $= 5x^2(x^2 - 9)$
$= 5x^2(x - 3)(x + 3)$

(ii) $6x^2 - 11x - 10$ $= (3x + 2)(2x - 5)$

(iii) $x^3 - 2x^2 - 9x + 18$ $= (x^3 - 2x^2) - (9x - 18)$
$= x^2(x - 2) - 9(x - 2)$
$= (x - 2)(x^2 - 9)$
$= (x - 2)(x - 3)(x + 3)$

(iv) $(1-x)^{\frac{1}{2}} - (1-x)^{-\frac{1}{2}}$ $= (1-x)^{-\frac{1}{2}}\left((1-x)^{1} - 1\right)$
$= (1-x)^{-\frac{1}{2}}(-x) = \dfrac{-x}{(1-x)^{1/2}}$

Q.11 (15 points) Perform the indicated operations and ***simplify*** as much as possible.

(i) $(2x-y)(x+5y)-3(x-y)^2$ $= (2x^2+10xy-5y^2) - 3(x^2-2xy+y^2)$ $-xy$

$= 2x^2+9xy-5y^2-3x^2+6xy-3y^2$

$= \boxed{-x^2+15xy-8y^2}$

(ii) $\left(\frac{x+1}{2x+3} \div \frac{x^2-4x-5}{2x^2+x-3}\right) - \frac{2}{x-5}$ $= \left(\frac{x+1}{2x+3} \cdot \frac{2x^2+x-3}{x^2-4x-5}\right) - \frac{2}{x-5}$

$= \left(\frac{x+1}{2x+3} \cdot \frac{(2x+3)(x-1)}{(x-5)(x+1)}\right) - \frac{2}{x-5}$

$= \frac{x-1}{x-5} - \frac{2}{x-5} = \frac{x-1-2}{x-5} = \boxed{\frac{x-3}{x-5}}$

(iii) $\frac{5}{x^2+x} - \frac{10}{x^2-1}$ $= \frac{5}{x(x+1)} - \frac{10}{(x-1)(x+1)}$

LCD $= x(x-1)(x+1)$

$= \frac{5(x-1)}{x(x-1)(x+1)} - \frac{10 \cdot x}{x(x-1)(x+1)}$

$= \frac{5x-5-10x}{x(x-1)(x+1)} = \frac{-5x-5}{x(x-1)(x+1)} = \frac{-5(x+1)}{x(x-1)(x+1)}$

$= \boxed{\frac{-5}{x(x-1)}}$

(iv) $(13x^4-8x^3+2x^2)-(5x^4-3x^3+2x^2-6)$

$= \boxed{8x^4-5x^3+6}$

(v) $\frac{(2x-7)^5}{(2x-7)^3} = (2x-7)^2 = \boxed{4x^2-28x+49}$

Q.12 (20 points) Solve each of the following equations. (***Without using a Calculator)***
(Show your steps)

(i) $2x-3(x-1)=5(3-x)+8$

$2x-3x+3=15-5x+8$ (1) for expanding ( )
$-x+3=23-5x$ (1) for simplifying each side
$\frac{4x}{4}=\frac{20}{4}$ (1) x's on one side/num.
$x=5$ (1) ÷ by 4 to get $x=5$ — {5}

(ii) $5x^2-6x=4x^2+6x-4$

(1) for all terms 4th: $x^2-12x+4=0$
(1) for comp □: $x^2-12x+6^2=-4+6^2$
(1) for writing $\sqrt{(\ )^2}$: $\sqrt{(x-6)^2}=\sqrt{32}$
(1/2): $x-6=\pm 4\sqrt{2}$
(1/2): $x=6\pm 4\sqrt{2}$

or

$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(1)(4)}}{2(1)}$
$=\frac{12\pm\sqrt{144-16}}{2}$ (1)
$=\frac{12\pm 8\sqrt{2}}{2}$ (1/2)
$=6\pm 4\sqrt{2}$ (1/2)

(iii) $2(2x+8)^2=54$

$\sqrt{(2x+8)^2}=\sqrt{27}$ (1) ÷ by 2
$2x+8=\pm 3\sqrt{3}$ (1/2) (1) for SQRT
$\frac{2x}{2}=\frac{-8\pm 3\sqrt{3}}{2}$ (1/2) for moving 8
$x=\frac{-8\pm 3\sqrt{3}}{2}$ (1) for dividing by 2

(iv) $x^4-2x^2+1=0$

(1) $(x^2-1)(x^2-1)=0$
$x^2-1=0$ or $x^2-1=0$
(1) $(x-1)(x+1)=0$ $x^2=1$
(1) $x=1$ or $x=-1$ $x=\pm 1$
{−1, 1}

(v) $\sqrt{2x+19}-8=x$

(1/2) for moving 8: $(\sqrt{2x+19})^2=(x+8)^2$
(1/2) for SBS: $2x+19=x^2+16x+64$ (1) for expansion
(1) move everything to RHS: $0=x^2+14x+45$
$0=(x+9)(x+5)$
$x=-9$ $x=-5$ (1) factors
$\sqrt{9}-8=-5$ ✓
Check $\sqrt{1}-8\neq -9$ rejected (1/2) rejected
{−5}

Q.13 (5 points) Solve the following absolute value equation..

$6|2x-1|-8=10$

$\frac{6}{6}|2x-1| = \frac{18}{6}$ ① for dividing by 6

$|2x-1| = 3$

① $2x-1 = 3$ or ① $2x-1 = -3$ ① + −

$\frac{2}{2}x = \frac{4}{2}$ $\frac{2}{2}x = -\frac{2}{2}$ $\{-1, 2\}$

$x = 2$ $x = -1$

① solve ① solve

Q.14 (10 points) Solve each of the following inequalities and ***graph*** the solution set on a number line. Express the solution set using ***interval notation.***

(i) $3 < 1+2x \leq 7$

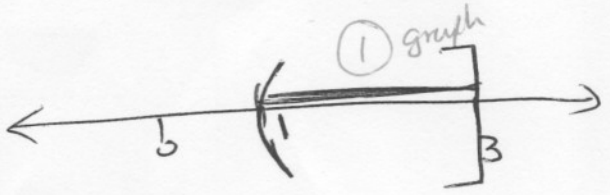
$-1 \quad -1 \quad -1$

① for many $\frac{2}{2} < \frac{2}{2}x \leq \frac{6}{2}$

① for ÷2. $1 < x \leq 3$

① for

① graph



$(1, 3]$ ① IN

$-7 \geq 3x-5 \geq 7$

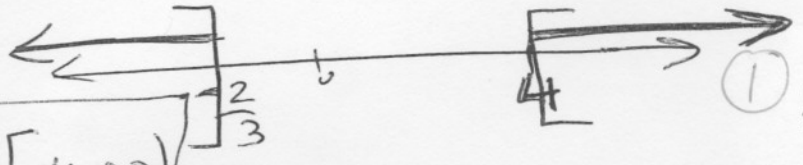
(ii) $\frac{2}{2}|3x-5| \geq \frac{14}{2}$

① for ÷ by 2 $|3x-5| \geq 7$

① $3x-5 \geq 7$ or $3x-5 \leq -7$ ①

$\frac{3}{3}x \geq \frac{12}{3}$ $\frac{3}{3}x \leq -\frac{2}{3}$ ↓ solve.

solve $x \geq 4$ $x \leq -\frac{2}{3}$



① graph

$(-\infty, -\frac{2}{3}] \cup [4, \infty)$ ① IN