PRINCE SULTAN UNIVERSITY

Business Calculus --- MATH 211

Second Major Exam

NAME _____ ID _____

Q1. Suppose the total cost in dollars of manufacturing q units of a certain commodity is $C(q) = 3q^2 + q + 500$.

a) Use marginal analysis to estimate the cost of manufacturing the 60^{th} unit.

b) Compute the actual cost of manufacturing the 60^{th} unit.

c) If the current level of production is 4 units and the manufacturer is planning to increase this to 4.1 units, estimate how the total cost will change as a result.

Q2. When the price of a certain commodity is p dollars per unit, customers demand x units of the commodity, where

$$x^2 + 3px + p^2 = 79.$$

How fast the demand changing with respect to time (t) when the price is 5 dollars per unit and is decreasing at the rate of 0.3 dollars per month?

Q3. Find the equation of the tangent line to the curve $x^2y^3 - 2xy = 6x + y + 1$ at the point (0, -1).

Q4. Find the critical numbers of the function

$$f(x) = 2x^3 + 6x^2 + 6x + 5$$

and determine their nature.

Q5. Sketch the graph of the function $f(x)$ that has the following properties

- $f'(x) > 0$ for $-2 < x < 0$ and $x > 2$.
- $f''(x) > 0$ for $x < -1$ and $x > 1$
- $f(0) = 3$

Q6. A certain machine depreciates so that its value after t years is $Q(t) = 10,000e^{-0.5t}$ dollars. At what rate is the value of the machine changing with respect to time after 3 years?

Q7. Consider the function $f(x) = x^3 + 3x^2 - 4$. Answer the following questions

a) Find the intervals of increasing and decreasing
b) Find the relative maximum and minimum points.
c) Study the concavity of the above function.
d) Use the above information to sketch the graph of the function.

Q6. A manufacturer can produce radios at a cost of 5 dollars apiece and estimates that if they are sold for x dollars apiece, consumers will buy 20-x radios a day. At what price should the manufacturer sell the radios to maximize profit?

Q7. Suppose that the demand function for a certain commodity is $q = 200 - 2p^2$ (for $0 \le p \le 10$).

a) Express the elasticity of demand as a function of p.

b) Calculate the elasticity of demand when the price is $p = 6$. Interpret your answer.

c) At what price is the elasticity of demand equal to -1?

Q8. At what interest rate, compounded continuously, should $3500 be invested today so that 10 years from now the account will be worth $7000?