



Prince Sultan University
MATH 221
First Examination
First Semester 2015/2016, Term 151
Dr. Ahmed Kaffel

Name: _____

ID Number # _____

Q1. (5 points) We consider the following machine number

01 0000000010 1011010000100000000.....00

Find the decimal number that represents this machine number.

Q2. (6 points)

We consider the function $f(x) = \frac{3}{x} + 2x$ with points $x_0 = \frac{1}{2}$, $x_1 = 1$, $x_2 = \frac{3}{2}$, $x_3 = 2$ and $x_4 = \frac{5}{2}$. Find the cubic Lagrange polynomial for the approximation of $f(1.2)$ and find the relative error.

Q3. (6 points) Use the bisection method to find the third iteration of the root of the equation

$x^3 - 3x - \frac{1}{2} = 0$ in the interval $[1, 2]$ and find a bound of the estimated absolute error.

Q4. (13 points) We consider the following iterative sequences:

a) $x_{n+1} = g_1(x_n)$

b) $x_{n+1} = g_2(x_n)$

where g_1 and g_2 are given by :

$$g_1(x) = \frac{1}{2}(x^2 - 3) \quad \text{and} \quad g_2(x) = \sqrt{2x + 3}$$

- 1) Find the values of the fixed points of the functions g_1 and g_2
- 2) Which of the following iterations is suitable to find the root of the equation $x^2 - 2x - 3 = 0$ in the interval $[2, 4]$.
- 3) Estimate the number of iterations required to achieve 10^{-3} accuracy using the fixed-point method, taking $x_0 = \frac{5}{2}$.

Q5. (3 points) Find the multiplicity of the root $\alpha = 1$ of the equation $(x - 1)\ln(x) = 0$