Prince Sultan University

MATH 223

Class Major Test II

Semester II, Term 162

Time Allowed: **90** minutes

Q1. (10 Points) Find a basis for the solution space of homogeneous linear system and find the dimension of that space.

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 3x_4 &= 0 \\ x_1 + 2x_2 - x_3 + 2x_4 &= 0 \\ x_2 + 3x_3 + 3x_4 &= 0 \end{aligned}$$

Q2. (6 Points) Let $T_1(x, y) = (x + y, x - y)$, $T_2(x, y) = (3x, 2x + 4y)$ be operators on R^2.

Find the following

a) $[T_1 \circ T_2]$
b) $[T_1^{-1}]$

Q3. (12 Points) Consider the matrix $A = \begin{bmatrix} 5 & 1 & 3 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

a) Find the characteristic equation of A.
b) Find the eigenvalues of A.
c) Find bases for the eigenspaces of A.

Q4. (12 Points) Consider two competing species that live in the same forest and let $x_1(t), x_2(t)$ denote the respective populations of the species at time t. Suppose the initial populations are $x_1(0) = 500, x_2(0) = 200$. If the growth rates of the species are given by

$$x_1'(t) = -3x_1(t) + 6x_2(t)$$

$$x_2'(t) = x_1(t) - 2x_2(t)$$

What is the population of each species at time t?

Q5. (10 Points) Let $u = (u_1, u_2), v = (v_1, v_2)$ be vectors in R^2.

Define the operation

$$< u, v > = 3u_1v_1 + 5u_2v_2$$

a. Show that this operation is an inner product.
b. If u = (2, - 1), v = (- 2, 3). Evaluate the following
   i. $< 2u - 3v, 3v >$
   ii. $\|u + v\|$
c. Find the matrix that generates the inner product.