

- 1) a) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - \tan^2 y}{4x^2 + y^2}$ does not exist.
- b) Find the absolute maximum and absolute minimum for the function $z = f(x, y) = x + y - xy$ in the closed triangular region D with vertices $(0,0)$, $(0,2)$, and $(4,0)$.

- 2) For the function $f(x, y) = x^3 - 12xy + 8y^3$, find the local maximum and minimum values and saddle points if any exist.

- 3) Consider the function $z = f(x, y) = \sqrt{25 - x^2 - y^2}$.
- a) Find the domain and range of $f(x, y)$, and sketch it.
 - b) Find $\frac{\partial z}{\partial x \partial y}$ at the point $(1, 1)$.

- 4) a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^4 + y^4 - 8xyz = 10$.
- b) If $z = f(x^2 - y^2)$, show that $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$.

5) a) Find the equation of the tangent plane to the graph of $f(x, y) = \sqrt{xy + e^{4y}}$ at the point $P(2, 0)$. Then, find the linear approximation to $f(2.1, 0.1)$.

b) Find the point on the cone $z = \sqrt{x^2 + y^2}$ that is closest to the point $P(1, 2, 0)$.