Prince Sultan University

Department o f Mathematical Sciences

Final Exam

Semester I, 2015 FALL (151)

26th December, 2015

MATH 211 – Business Calculus

Time Allowed : 120 minutes (2 Hours)
Maximum Points: 100 points

Name of the student: _____

ID number : _____

Section: 125

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 7 pages and 16 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	*Your Score*
Q.1 , Q.2	22	
Q.3 , Q.4	15	
Q.5 , Q.6 , Q.7	16	
Q.8 , Q.9 , Q.10 , Q.11	19	
Q.12 , Q.13 , Q.14	16	
Q.15 , Q.16	12	
Total	**100**	

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Q.1 (16points): Find the derivative, $\frac{dy}{dx}$: **(Simplify)**

a) $y = \ln\sqrt{2x^2+3}$

b) $y = x^3(1-3x)^5$

c) $y = x^2e^{-x}(3x+5)^3$

d) $y = \left(1+\frac{5}{4x}\right)^4$

Q.2 (6 points): Find the equation of the tangent line to the curve $x^3 + x\cdot y + y^3 = x$ at the point $(1,0)$

Q.3 (12 points): Evaluate the integrals:

a) $\int 12x\sqrt{1-3x^2}\,dx$

b) $\int\left(\frac{3e^{-x}+2e^{3x}}{e^{2x}}\right)dx$

c) $\int_1^e x^3\ln(x)\,dx$

Q.4 (3 points): Solve for x if $\frac{25e^x}{e^x+3}=10$

Q.5 (4 points): Find the value of k for which $f(x)=\begin{cases} x^2+1 & x\geq -2 \\ 3x+k & x<-2 \end{cases}$ is continuous at $x=-2$

Q.6 (6 points): Find the area bounded by the graphs of $f(x)=x^2-4x$ and $g(x)=x-4$

Q.7 (6 points): Given the function $f(x)=2x^4-4x^2-1$

a) Find the critical points.
b) Find the intervals of increase and decrease.
c) Find the relative maximum and minimum, if any. **Don't Graph**

Q.8 (6 points): Find the absolute maximum and absolute minimum of $f(x) = x + \frac{1}{x}$ in $\left[\frac{1}{2}, 3\right]$

Q.9 (4 points): Find the intervals on which the graph $f(x) = -x^3 + 3x^2 - 3x - 9$ is concave up and concave down.

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Q.10 (5 points): A manufacturer estimates when x units of a certain commodity are produced each month, the total cost will be $C(x) = x^3 + 5x + 162$ dollars, and all x units can be sold at the price $p(x) = 180 - 2x$ dollars per unit. Determine the level of production that results in maximum profit. What is the maximum profit?

Q.11 (4 points): The marginal revenue of producing q units of a certain commodity is $R'(q) = q(10 - q)$ hundred dollars per unit. How much revenue is generated as the level of production is increased from 4 to 9 units?

Q.12 (6 points): Suppose q units of a certain commodity are demanded by consumers when the price, p is $D(q) = 100 - 2q - 3q^2$ dollars per unit. What is the **consumer's surplus** for the commodity when the level of production is $q_0 = 5$ units?

Q.13 (6 points): Suppose that $q = 60 - 0.1p$ units of a certain commodity are demanded when p dollars per unit are charged $(0 \le p \le 600)$. Find the elasticity of demand when the price is $p = 200$. Should the manufacturer raise or lower this price in order to increase total revenue? Explain.
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Q.14 (4 points): After t months on the job, a postal clerk can sort $Q(t) = 700 - 400e^{-0.5t}$ letters per hour. What is the average rate at which the clerk sorts mail during the first 4 months on the job?

Q.15 (6 points): A manufacturer's total cost is $C(q) = q^3 - 5q^2 + 500q + 200$ dollars, where q is the number of units produced.

a) Use marginal analysis to estimate the cost of producing the 10^{th} unit.

b) Find the actual cost of producing the 10^{th} unit.

Q.16 (6 points): Compute the partial derivatives, f_x , f_y , f_{xx} , f_{yy} , f_{xy} , and f_{yx} .

$f(x, y) = 3x^5 y - 4x + ye^x$