PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences
MATH 211 – Business Calculus
Final Examination
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Maximum Time 150 Minutes

Q1. (a) Evaluate the limit $\lim_{x\to 2} f(x)$, where $f(x)=\begin{cases} 2x^2-1 & if \quad x<2 \\ x+1 & if \quad x\geq 2 \end{cases}$

(b) Find the slope of the tangent line to the curve $x^2y^3-e^{x^2}+5y=4$ at the point (0, 1)

(c) The average property tax in a certain community was $T(x)=25x^2+40x+800$ dollars where x is number of years after 1998.

i. At what rate was the property tax increase with respect to time in 2001?

ii. At what percentage rate was the property tax increase with respect to time in 2001?

Q2. (a) Suppose $D(p)=200-p^2$ units is the demand function for a certain commodity. Compute the elasticity of the demand at the price p = 10 dollars per unit and determine whether the demand is elastic, inelastic or of unit elasticity at the indicated price.

(b) A manufacturer estimates that when q units of a certain commodity are produced each month, the total cost will be $C(q)=q^3+5q+162$ dollars, and all q units can be sold at the price $p(q)=180-2q$ dollars per unit. Determine the level of production that results in maximum profit. What is the maximum profit?

(c) Sketch the graph of the function $f(x)=3x^4-4x^2+3$.

(d) Find the derivative of the functions

I. $y=\sqrt{x^2-5x+1}$

II. $y=\ln\left(\dfrac{x+3}{x^2-5}\right)$

Q3. Evaluate the following integrals

a) $\int_{-2}^{2} (x^5 + 3x^3 - 2x)\,dx$

b) $\int \frac{(x-3)^2}{x} dx$

c) $\int 3xe^{2x^2-1} dx$

d) $\int_{0}^{1} \frac{x\,dx}{(3x^2-7)^2}$

Q4. (a) Determine the area of the region bounded by the curves $y = 7 + 4x - x^2$ and $y = 7 + x^2$.

(b) Find the average value of $f(x) = x^2 - 3x + 5$ over the interval $1 \le x \le 3$. (c) Suppose a study indicates that the distribution of income for professional football players and basketball players are given by the Lorentz curves $L_1(x) = \frac{5}{6}x^2 + \frac{1}{6}x$ and $L_2(x) = \frac{3}{5}x^4 + \frac{2}{5}x$, respectively. Find the **Gini index** for each professional sport and determine which has the most equitable income distribution.

(d) The demand function for a certain commodity is $D(q) = 32 - 2q^2$ dollars per unit, where q is the level of production.

(i) Find the total amount of money consumers are willing to spend when $q = 5$ units.

(ii) If the supply function is $s(q) = \frac{1}{3}q^2 + 2q + 5$, find the consumers surplus at the equilibrium price.