# Prince Sultan University
# Orientation Mathematics Program

MATH 223

Final Exam

Semester II, Term 162

Sunday, May 21st , 2017

Time Allowed: **180** minutes

Q1. (8 Points) Find the shortest distance between the point (1, 2, 1) and the plane $2x - 3y + z = 2$

Q2. (8 Points) Find the area of the triangle with the vertices $(1,1), (2,2), (3,-3)$.

Q3. (8 Points) Show that if $\lambda$ is an eigenvalue of a matrix A, then $\lambda^2$ is an eigenvalue of $A^2$.

Q4. (15 Points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

a) Find the eigenvalues and the corresponding eigenvectors.
b) Determine whether the matrix A is diagnalizable or not. (Explain)
c) Use the diagnolization of A to calculate $A^8$.

Q5. (10 Points) Solve the differential equation system

$$\begin{aligned} y_1' &= 2y_1 - 3y_2 \\ y_2' &= -4y_1 + y_2 \end{aligned}$$

Q6. (8 Points) Find the least squares straight line fit y= ax + b to the four points (0, 2), (1, 6), (2, 8), (2, 10).

Q7. (10 Points) Let $W$ be the span of the linearly independent vectors $v_1 = (1,1,1,1), v_2 = (2,1,0,1), v_3 = (1,1,2,1)$ in $R^4$.

a) Apply Gam-Schmidt to find an orthonormal basis for W.
b) Write the vector $v = (2,3,5,3)$ as a linear combination of the orthonormal basis.

Q8. (13 Points) Let $P_2$, the set of all polynomials of degree less than or equal to 2, have the inner product

$$< p, q >= \int_{-1}^{1} p(x)q(x)dx$$

Let $p = 5x, q = -3x^2 + 1$.

a) Evaluate $< xp, 2q - p >$
b) Determine whether the vectors $p$ and $q$ are orthogonal or not.
c) Verify the equation $\|p\|^2 + \|q\|^2 = \|p + q\|^2$
d) Evaluate $d(1, p)$
e) Show that the subset $W = \left\{p(x) \in P_2 \text{ such that } \int_{-1}^{1} p(x)dx = 0\right\}$ is a vector subspace of $P_2$