1) a) If $z = f(x, y)$ has continuous second-order partial derivatives and $x = r^2 + rs$ and $y = 3rs$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial^2 z}{\partial r^2}$. Then, apply your result to evaluate $\frac{\partial^2 z}{\partial r^2}$ when $r = 1$ and $s = 0$ for the function $z = f(x, y) = xe^{xy}$.

b) Given that the function $g(x, y) = y^2 - 2y\, cosx$ has the critical points $A(0,1), B(\pi, -1), C(2\pi, 1), D\left(\frac{\pi}{2}, 0\right), E(\frac{3\pi}{2}, 0)$ on the interval $[-1,7]$, find the local maximum and minimum values and saddle point(s) of g if they exist.

2) Evaluate the integrals

a) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$

b) $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dxdy$

c) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2} dzdydx.$

3) Consider the vectors $u = (2, -1, 3)$, $v = (1, -3, 4)$ and $w = (2, 2, 1)$.

a) Find the norm of $u - 2v$ and the distance between u and w.

b) Find a vector which is orthogonal to both u and v.

c) Calculate the scalar triple product $u.(v \times w)$. What is the volume of the parallelepiped in $3-$space determined by the vectors u, v and w.

d) Find the cosine of the angle between the vectors u and w.

e) Find the vector component of u along v and the vector component of u orthogonal to v.

4) Use Gram-Schmidt process to find the $QR - decomposition$ of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$.

5) a) Show that eigenvectors of a symmetric square matrix, taken from different eigenspaces are orthogonal.

b) Let $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.

i) Find the eigenvalues and eigenspaces for the matrix A.

ii) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

iii) Find the eginvalues and eigenspaces for the matrix $B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.