PRINCE SULTAN UNIVERSITY

MATH 111 Calculus

FINAL EXAMINATION

Start : 8:30 AM
End: 11:00 AM

Name _____

I.D. _____

Section: **(8 AM)** **(10 A.M)** **(11 AM)** **(12 PM)**

1. Answer all questions
2. This exam consists of **7 pages, 13 questions**
3. You can use a calculator, NOT a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1, 2	**16**	
3	**15**	
4, 5, 6	**18**	
7, 8, 9	**21**	
10	**12**	
11, 12, 13	**18**	
TOTAL	**100**	

Q1. (10 points) Find the limits

$$\lim_{x\to 1}\frac{x^{2014}-1}{x^{2000}-1}$$

$$\lim_{x\to 0^+}\sqrt{x}e^{\sin(\frac{\pi}{x})}$$

$$\lim_{x\to\infty}\frac{ax+b}{\ln(c+e^x)}$$

Q2. (6 points) Find all values of the constant c that make the function $f(x)$ continuous on $(-\infty, \infty)$

$$f(x)=\begin{cases} c^2x^2-3x-1 & if\ x<1 \\ 3c\cos(x-1) & if\ x\geq 1 \end{cases}$$

All work must be shown.

Q3. (15 points) Find the derivative $\frac{dy}{dx}$ of the functions (Simplify your answers if possible)

a. $y = \sqrt{\frac{x^2+1}{x^2+4}}$

b. $y = e^{\cos x} + \cos(e^x)$

c. $y = \ln\left(\frac{x^5\sqrt{x^2-1}}{(x-3)^5\sin(x)}\right)$

d. $y = log_{2x}(5) + log_5(2x)$

e. $y^x = x^y$

Q4. (7 points) Find equations of the tangent line to the curve $x^3 + 3x^2y^2 + 5y^3 + y = 8$ at the point $(2,0)$.

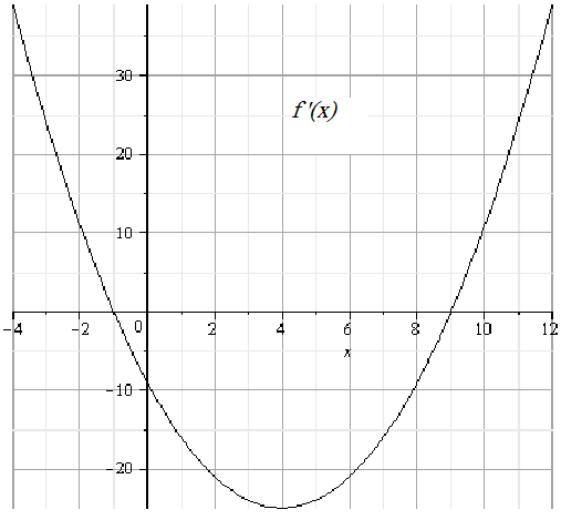
Q5. (6 points) A spherical balloon is losing air at a rate of $10\,\pi\ cm^3/min$. At what rate is its radius r decreasing when $r = 2cm$?

Q6. (5 points) Find the derivative of the function $f(x) = \sinh^{-1}(\tan x)$

Q7. (6 points) Find the absolute maximum and the absolute minimum of $f(x) = \frac{x}{x^2+1}$ on the interval [0, 3].

Q8. (7 points) Verify that the function $f(x) = x^3 + x - 1$ satisfies the hypothesis of the Mean Value Theorem on the interval [0, 2]. Then find all numbers *c* that satisfy the theorem.

Q9.(8 points) The graph of the derivative $f'(x)$ of a continuous function $f(x)$ is shown below



a) On what interval is f increasing?

b) At what value of x does f have local maximum and local minimum

c) On what interval is f concave up?

d) State the *x*-coordinate(s) of the inflection point(s)

Q10. (12 points) Graph the function $f(x) = x + \frac{1}{x}$ by following the following steps:

a) Find the domain.

b) Find the *x* and *y*-intercepts, if any.

c) Find the Asymptotes if any.

d) Find the intervals on which *f* is increasing and the intervals on which *f* is decreasing and the local minimum and local maximum values, if any.

e) Find the intervals on which *f* is concave up and the intervals on which *f* is concave down and the inflection points (if any).

f) Sketch the graph of *f*.

Q11. (5 points) Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

Q12. (6 points) If $f(2) = 10$ and $f'(x) = x^2 f(x)$ for all x, find $f''(2)$.

Q13. (7 points) A farmer wants to fence an area of 15000 m^2 in a rectangular field and then divide it in half with a fence parallel to one of sides of the rectangle. How can he do this to **minimize** the cost of the fence?