

PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences
MATH 211 – Business Calculus

Final Examination

June 2011

Start: 1:00 PM

End: 3:30 PM

Q1. (a) **(4 points)** Let $f(x) = \begin{cases} 3 - 2x & \text{if } x \leq -1 \\ x^2 - x + 3 & \text{if } x > -1 \end{cases}$

- i. Find the limit, $\lim_{x \rightarrow -1} f(x)$.
- ii. Is the function continuous at $x = -1$? (Explain)

(b) **(10 points)** Use calculus to sketch the graph of the function

$$f(x) = x^4 + 2x^3 - 12x^2.$$

(c) **(6 points)** A manufacturer's total revenue is $R(q) = 240q - 0.05q^2$ dollars where q units are produced. Currently, the manufacturer is producing 80 units and is planning to decrease the output by 0.65 units.

- i. Estimate how the total revenue will change as a result.
- ii. Estimate the percentage change as a result.

Q2. (a) **(4 points)** Find the tangent line to the curve $xy^2 - 3xy + 5x - 2y = 4$ at the point $(0, -2)$

(b) **(4 points)** How long will it take \$5000 to grow to \$7500 if the interest rate is 7% compounded quarterly?

(c) **(6 points)** Find the derivative of the following functions:

- i. $y = \frac{x^2}{x^2 - 6}$
- ii. $y = \frac{\ln(x+1)}{x+1}$
- iii. $y = x^2 \sqrt{2x + 5}$

Q3. (a) **(8 points)** The average cost of producing x units of a particular commodity is $AC(x) = x^2 - 24x + 350 + \frac{338}{x}$ dollars per unit.

Determine the level of production x for which the average cost is minimized.

(b) **(8 points)** At a certain factory, when K thousand dollars is invested in the plant, the production Q is changing at a rate of $Q'(K) = 300K^{-0.6}$ units per thousand dollars. When \$5500 is invested, the level of production is 3200 units.

How many units must be produced when \$24000 are invested?

(c) **(12 points)** Evaluate the following integrals

- i. $\int_1^3 3(2x^2 - 4)^2 dx$
- ii. $\int \frac{e^x}{e^x - 2} dx$
- iii. $\int \frac{3x+6}{\sqrt{x^2+4x-1}} dx$

Q4. (a) **(8 points)** Suppose that the consumers' demand function for a certain commodity is $D(q) = 50 - 3q - q^2$ dollars per unit.

Compute the consumers' surplus to get 3 units of the commodity.

(b) **(8 points)** Find the area of the region bounded by the curves $y = \frac{4}{x}$ and $y = 5 - x$.

(c) **(6 points)** A bicycle manufacturer expect that x months from now will be buying 500 bicycles per month at the price of $p(x) = 200 + 3\sqrt{x}$ dollars per bicycle. What is the **average revenue** the manufacturer can expect from the sales of the bicycles over the first 16 months?

Q5. (a) **(4 points)** Let $f(x, y) = \frac{4xy - 3x^2 - 5y}{\sqrt{y - x^2 + 2}}$.

Find the domain and graph it.

(b) **(6 points)** Let $f(x, y) = \ln(x^2 - y)^4$.

- i. Find the first-order partial derivatives, f_x and f_y .
- ii. Show that $x f_x + 2y f_y = 8$.

(c) **(6 points)** A sport store in Riyadh carries two types of sport Jerseys, Al-Hilal Jerseys and Al-Naser jerseys. Sales figure indicate that if Al-Hilal jerseys sells for x SR per item and Al-Naser jerseys sells for y SR per item, the demand for Al-Hilal will be $D_1 = 300 - 20x + 30y$ jerseys and the demand for Al-Naser will be $D_2 = 400 + 40x - 10y$ jerseys.

- i. Express the store's total revenue as a function of the prices x and y .
- ii. Find the marginal revenue of Al-Hilal and the marginal revenue of Al-Naser when Al-Hilal jerseys sells for SR 20 per jerseys and Al-Naser jerseys sells for SR 24 per jerseys.