Prince Sultan University

Department of Mathematical Sciences

MATH 223 – Second Examination

8th May 2007

Dr. Aiman Mukheimer

Student Name: ______________________________ **Student ID:**_________________

Time allowed: 90 minutes **Maximum points: 100 points**

1. (8 points) Find the coordinate vector of $p = 2 - x + x^2$ relative to the basis $S = \{p_1, p_2, p_3\}$ where $p_1 = 1 + x$, $p_2 = 1 + x^2$, and $p_3 = x + x^2$

2. (8 points) Use the Wronskian to show that the vectors e^x, xe^x, and x^2e^x are linearly independent

3. (10 points) Check whether the following vectors $v_1 = (0,3,1,-1)$, $v_2 = (6,0,5,1)$, and $v_3 = (4,-7,1,3)$ form a linearly dependent set in R^4. If so Express any vector as a linear combination of the other two.

4. (10 points) Determine whether the following vectors $v_1 = (2,-1,3)$, $v_2 = (4,1,2)$, and $v_3 = (8,-1,8)$ span the vector space R^3

5. (9 points) Show that the set of all positive real numbers with operations
$x + y = xy$ and $kx = x^k$ is a vector space

6. (8 points) Determine whether $T : R^3 \to R^2$ where $T(x, y, z) = (3x - 4y, 2x - 5z)$ is a linear transformation or not.

7. (15 points) Show that the lines

$$x = 3 - 2t\ , \quad y = 4 + t\ , \quad z = 1 - t \qquad (-\infty < t < \infty)$$

and $$x = 5 + 2t\ , \quad y = 1 - t\ , \quad z = 7 + t \qquad (-\infty < t < \infty)$$

are parallel, and find an equation for the plane they determine.

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8. (5 points) For which values of k are $u = (k, k, 1)$ and $v = (k, 5, 6)$ are orthogonal.

9. (10 points) Find the point of intersection of the line
$x-9=-5t$, $y+1=-t$, $z-3=t$ $(-\infty<t<\infty)$ and the plane $2x-3y+4z+7=0$

10. (9 points) Find the standard matrices for linear operators on R^3 represented by:

a. A reflection about the yz-plane.

b. An orthogonal projection on the xz-plane.

c. The composotion of a reflection about the yz-plane, followed by an orthogonal projection on the xz-plane.

11. (8 points) Show that the liear operator $T : R^2 \to R^2$ defined by the equations $w_1 = 2x_1 + x_2$ and $w_2 = 3x_1 + 4x_2$ is one-to-one, and find $T^{-1}(w_1, w_2)$