Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

Statistics and Probability Theory	STAT 101	Final Exam
Semester:	Fall Semester --Term 181	
Date:	December 17, 2018	
Time Allowed:	**180 minutes**	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section/Time	
Instructor's Name:	Dr. Bahaa Abdalla Dr. Eric Benson Dr. Mohammed Kaouache

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 2	Page 3	Page 4	Page 5	Page 6				Page 7	Total
Questions	**15**	**15**	**14**	**12**	**12**				**12**	**80**
Marks										

15 points

1. You are given the following parameters for two populations:

Population 1	Population 2
$\mu = 700$	$\mu = 29000$
$\sigma^2 = 300$	$\sigma = 5000$

a. Compute the coefficient of variation for each population. **(4 points**)

b. Based on the answers to part (a), which population has data values that are more variable relative to the size of the population means? **(2 points)**

c. What is the minimum proportion of observations in population 2 that are in the range 6500 to 51500? **(4 points)**

d. If population 2 is normally distributed, find the probability that a value randomly selected from this population will have a value between 15150 and 21650. **(4 points)**

e. If population 1 follows a uniform distribution, determine the values of *a* and *b*. **(4 points)**

2. Three events occur with probabilities

$$P(E_1) = 0.35, \qquad P(E_2) = 0.25, \qquad P(E_3) = 0.4.$$

other probabilities are

$$P(B|E_1) = 0.25, \qquad P(B|E_2) = 0.15, \qquad P(B|E_3) = 0.6.$$

a. Compute $P(B)$. (3 points)

b. Compute $P(B \text{ and } E_1)$. **(3 points)**

c. Compute $P(B \text{ or } E_1)$. **(3 points)**

d. Compute $P(E_2|B)$. **(3 points)**

e. Assume that $E_1, E_2, \text{and } E_3$ are independent events, Calculate $P(E_1 \text{ and } E_2 \text{ and } E_3)$. **(3 points)**

3. Arrivals to a bank automated caller machine (ATM) are distributed according to a Poisson distribution with a mean equal to three per 15 minutes.

(a) (4 points) Determine the probability that in a given 15-minute segment, no customers will arrive at the ATM.

(b) (3 points) What is the probability that fewer than four customers will arrive in a 30-minutes segment?

(c) (3 points) What is the probability that at least 7 customers will arrive in the next hour?

(d) (3 points) What is the probability that the time between the next two customers will be 90 seconds or less?

(e) (3 points) What is the probability that the time between the next two customers will be greater than 10 minutes?

4. Suppose that a battery is expected to last for three years, the standard deviation of the life of this battery is 0.3, and the life of the battery is normally distributed.

a. Determine the probability that the battery will last longer than 3.5 years. **(3 points)**

b. Compute the length-of-life value for which 10% of the batteries last longer. **(3 points)**

c. If we select a random sample of 50 batteries, what is the probability that the sample mean will not exceed 3.2 years? **(4 points)**

d. If a sample of 20 batteries is selected and the sample mean is found to be 2.93, compute the sampling error for the sample mean. **(3 points)**

5. Given a population in which the probability of a success is 0.4. Take a sample of 1000.

a. Calculate the probability that the proportion of successes in the sample will be at most 0.42. (4 points)

b. Calculate the probability that the sample proportion will be within ± 0.05 of the population proportion. (4 points)

c. Based on the sample data, construct a 92% confident interval for the population proportion. (5 points)

6. **Carbohydrates** The following represents a random sample of the amounts of carbohydrates (in grams) for 20 sandwiches from a fast food restaurant. (Source: McDonald's Corporation)

31	33	34	33	37	40	40	45	37	38
44	51	59	52	60	54	62	39	33	26

a. Construct an 80% confidence interval for the population mean. **Interpret this result.**

b. Construct a 95% confidence interval for the population mean. **Interpret this result.**

7. In a manufacturing process, the error made in determining the composition of an alloy is a random variable having the uniform density with $a = -0.075$ and $b = 0.010$.

   a. What is the probability that such an error will be between 0.050 and 0.001?

   b. What is the probability that such an error will be between 0.001 and 0.008?

   c. Find the error that is less than 46% of all the errors.

   d. Find the interquartile range of this distribution of errors.

8. Suppose that, next month, the quality control division will inspect 30 units. Among these, 20 will undergo a speed test and 10 will be tested for current flow. If an engineer is randomly assigned 6 units, what are the probabilities that

    a. None of them will need a speed test?
    b. Only 2 will need a speed test?
    c. At least 3 will need a speed test?

9. A maker of specialized instruments receives shipments of 24 circuit boards. Suppose one shipment contains 4 that are defective. An engineer selects a random sample of size 4. What are the probabilities that the sample will contain
    a. 0 defective circuit boards?
    b. 1 defective circuit board?
    c. 2 or more defective circuit boards?

10. The time between breakdowns of aging machines is known to be exponentially distributed with a mean of 25 hours. The machine has just been repaired.
    a. Determine the probability that the next breakdown occurs more than 50 hours from now.
    b. Determine the probability that the next breakdown occurs one and a half to two days from now.
    c. Find the breakdown time that lies in the 95percentile of this distribution.
    d. What is the interquartile range of this distribution?

11. A survey report states that 70% of adult women visit their doctors for a physical examination at least once in two years. If 20 adult women are randomly selected, find the probability that

    a. Fewer than 14 of them have had a physical examination in the past two years.
    b. At least 17 of them have had a physical examination in the past two years.
    c. Find the mean and standard deviation of adult women who visit their doctors for a physical examination at least once in two years.
    d. Find the proportion of adult women who visit their doctors for a physical examination at least once in two years within **1.4** standard deviation of the mean.