Prince Sultan University
Department of Mathematics and Physical Sciences

Math 215
Final Examination
Semester I, Term 131
Tuesday, January 14, 2014

Time Allowed: 120 minutes

Name:

Student Number:

Important Instructions

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There must be NO talking during the examination.
5. This examination has 12 problems. Make sure your paper has all these problems.

Statement of Ethics:

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

Total Grade out of 80:

1. A) (2 points) Show that $\vec{u} = (a, b)$ and $\vec{v} = (-b, a)$ are orthogonal vectors.

   B) (3 points) Use the result in part (A) to find two vectors that are orthogonal to $\vec{w} = (2, -3)$.

2. (5 points) Find the area of the parallelogram determined by $\vec{u} = (2,3,0)$ and $\vec{v} = (-1,2,-2)$.

3. (6 points) Find an equation for the plane through $(-2,1,5)$ that is perpendicular to the planes $4x - 2y + 2z = -1$ and $3x + 3y - 6z = 5$.

4. (7 points) Determine whether the vectors $\vec{v}_1 = (2,2,2)$, $\vec{v}_2 = (0,0,3)$, and $\vec{v}_3 = (0,1,1)$ span R^3.

5. (6 points) Evaluate the limit if it exists, or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4 + 4y^2}$.

6. (15 points) Let $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$.

a) Find the eigenvalues of A.

b) Determine whether A is diagonalizable or not.

c) If so, find a matrix P that diagonalizes A and determine $P^{-1}AP$.

d) Find A^{11}.

7. (4 points) Find $\vec{u} \bullet \vec{v}$ given that $\|\vec{u} + \vec{v}\| = 1$ and $\|\vec{u} - \vec{v}\| = 5$.

8. (6 points) Find the volume of the solid under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

9. (8 points) Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2 y + 4$ on the set $D = \{(x, y) : |x| \le 1, |y| \le 1\}$.

10. (5 points) Change the order of integration $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\,dzdydx$ to spherical coordinates.

11. (6 points) Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the $xy-plane$.

12. (7 points) Evaluate $\iiint_E \frac{z}{x^2+z^2}\,dV$ where $E = \{(x,y,z): 1 \le y \le 4, y \le z \le 4, 0 \le x \le z\}$.

Hint: Use the relation $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C.$