Prince Sultan University

Department of Mathematics
and
Physical Sciences

Math 225
Major II
Term 141
Sunday, December 7, 2014

Time Allowed: 80 minutes

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Name:
Student Number:

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Statement of Ethics:
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

| Total/55: |
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| Total/22.5: |
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Q.1 (4+7 points) Consider the equation

$$x^2y'' - 3xy' + 4y = x^2, \quad x > 0. \tag{1}$$

a) Verify that the functions $y_1(x) = x^2$ and $y_2(x) = x^2 \ln x$ satisfy the corresponding homogeneous equation $x^2y'' - 3xy' + 4y = 0.$

b) Use variation of parameters method to find a particular solution of equation (1).

Q.2 (5+2+5 points) Consider the equations

$$y^{(4)} - 4y'' = 0\,, \tag{2}$$

and

$$y^{(4)} - 4y'' = t^2 + 4e^t\,. \tag{3}$$

a) Find the complementary solution of (2) with $y(0) = 0, \ y'(0) = 0, \ y''(0) = -2, \ y'''(0) = 0.$

b) How does the solution behaves as $t \to \infty.$

c) Use method of undetermined coefficients to determine a particular solution of equation (3). Do not evaluate the constants.

Q.3 (10 points) Consider the equation $y'' - xy' - y = 0.$ Find a power series solution of the differential equation about $x_0 = 0.$

Q.4 (5+4 points) Consider the equation

$$x^2y'' - 3xy' + 4y = 0. \tag{4}$$

a) Find a general solution of (4) of the form $x^r$.

b) Show that the Wronskian is nonzero.

Q.5 (5+8 points)

a) Show that the Laplace transform of $f(t) = e^{5t}$, is $F(s) = \dfrac{1}{s-5}, \ s > 5.$

b) Use Laplace transform to solve the initial value problem

$$y'' - y' - 6y = 0, \ y(0) = 2, \ y'(0) = -1.$$

Note: the equations in Q.1 and Q.4 are the same.