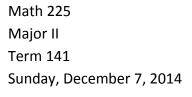
Prince Sultan University

Department of Mathematics and Physical Sciences





Time Allowed: 80 minutes
Name:
Student Number:
Statement of Ethics:
I agree to complete this exam without unauthorized assistance from any person, materials, or
device.
Signature:

Total/22.5:

Total/55:

Q.1 (4+7 points) Consider the equation

$$x^{2}y'' - 3xy' + 4y = x^{2}, x > 0.$$
 (1)

- a) Verify that the functions $y_1(x) = x^2$ and $y_2(x) = x^2 \ln x$ satisfy the corresponding homogeneous equation $x^2 y'' 3xy' + 4y = 0$.
- b) Use variation of parameters method to find a particular solution of equation (1).

Q.2 (5+2+5 points) Consider the equations

$$y^{(4)} - 4y'' = 0, (2)$$

and

$$y^{(4)} - 4y'' = t^2 + 4e^t. (3)$$

- a) Find the complementary solution of (2) with y(0) = 0, y'(0) = 0, y''(0) = -2, y'''(0) = 0.
- b) How does the solution behaves as $t \to \infty$.
- c) Use method of undetermined coefficients to determine a particular solution of equation (3). Do not evaluate the constants.

Q.3 (10 points) Consider the equation y'' - xy' - y = 0. Find a power series solution of the differential equation about $x_0 = 0$.

Q.4 (5+4 points) Consider the equation

$$x^2y'' - 3xy' + 4y = 0. (4)$$

- a) Find a general solution of (4) of the form x^r .
- b) Show that the Wronskian is nonzero.

Q.5 (5+8 points)

- a) Show that the Laplace transform of $f(t) = e^{5t}$, is $F(s) = \frac{1}{s-5}$, s > 5.
- b) Use Laplace transform to solve the initial value problem

$$y'' - y' - 6y = 0$$
, $y(0) = 2$, $y'(0) = -1$.

Note: the equations in Q.1 and Q.4 are the same.