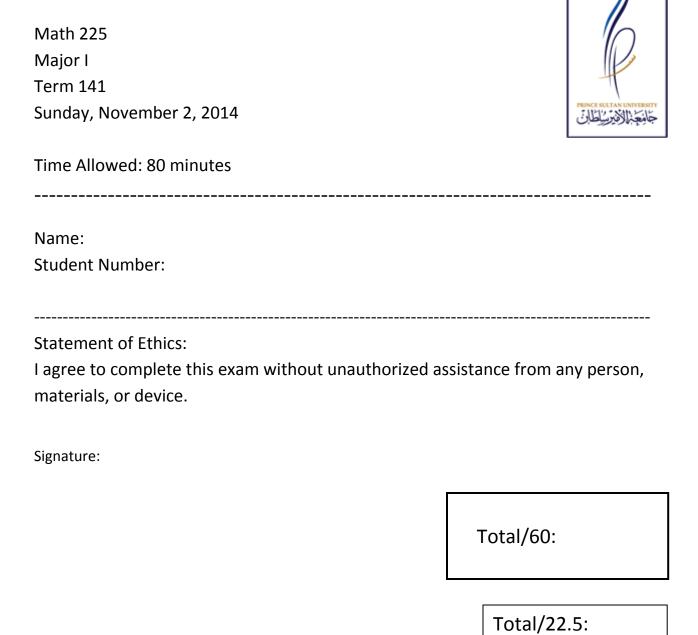
Prince Sultan University Department of Mathematics and Physical Sciences



I. Questions of First Order Differential Equations:

Q.1 (10 points) Given the following first order differential equations:

a)
$$y' = y^2 e^x$$
.

b)
$$y' = 1 + 2x + y + 2xy$$
.

c)
$$y' + 2y = 4x^2 - 2$$
.

d)
$$\cos x dy = (y \sin x + e^x + 2) dx$$
.

e)
$$y' = \sqrt{x} - y$$
.

f)
$$y' = x - \sqrt{y}$$
.

g)
$$y' = x^2 + y^2$$
.

h)
$$2x + y^2 + 2xyy' = 0$$
.

Classify them into the following categories:

- i. Separable Equations:-----
- ii. First Order Linear Differential Equations:.....
- iii. Exact Equations:....
- iv. Nonlinear Differential Equations:.....

Q.2 (10 points) Consider the initial value problem (IVP): $y' = \frac{2x}{1+2y}$, y(1) = 0.

- a) Find the solution of the IVP in explicit form.
- b) By looking at the explicit form of the solution, determine the interval in which the solution is defined.

- Q.3 (10 points) Consider the equation: $\left[4\left(\frac{x^3}{y^2}\right) + \left(\frac{3}{y}\right)\right] + \left[3\left(\frac{x}{y^2}\right) + 2y\right]y' = 0.$
 - a) Find an integration factor of the form $\frac{N_x M_y}{M}$.
 - b) Solve the differential equation.

II. Questions on Seconder Order differential Equations:

Q.3 Consider the differential equation xy'' - (x+1)y' + y = 0.

- a) Given that $y_1 = e^x$ is a solution of the differential equation, use method of reduction of order to find a second solution.
- b) Check the Wronskian of y_1 and y_2 and decide whether they form a fundamental set of solutions.
- Q.4 (10 points) Consider the differential equation xy'' (x+1)y' + y = 0.
 - c) Given that $y_1 = e^x$ is a solution of the differential equation, use method of reduction of order to find a second solution.
 - d) Check the Wronskian of y_1 and y_2 and decide whether they form a fundamental set of solutions.

Q.5 (8 points) Consider the differential equation: $y'' + 3y' = 2t^4 + t^2e^{-3t} + 2\sin 3t$. Write a form for the general solution y_g if the method of undetermined coefficients is to be used. <u>Do not</u> evaluate the constants.

Q.6 (12 points) Consider the initial value theorem (IVP)
$$\begin{cases} y'' + 2y' + 6y = 0, \\ y(0) = 2, \\ y'(0) = \alpha \ge 0. \end{cases}$$

- a) Find the solution of this problem.
- b) Find α such that y = 0 when t = 1.
- c) Determine the end behavior of the solution as $t \to \infty$.
