Prince Sultan University
Department of Mathematics and Physical Sciences

Math 225
Major I
Term 141
Sunday, November 2, 2014

Time Allowed: 80 minutes

Name:
Student Number:

Statement of Ethics:
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

Total/60:

Total/22.5:

I. Questions of First Order Differential Equations:

Q.1 (10 points) Given the following first order differential equations:

a) $y' = y^2 e^x$.

b) $y' = 1 + 2x + y + 2xy$.

c) $y' + 2y = 4x^2 - 2$.

d) $\cos x dy = (y \sin x + e^x + 2)dx$.

e) $y' = \sqrt{x} - y$.

f) $y' = x - \sqrt{y}$.

g) $y' = x^2 + y^2$.

h) $2x + y^2 + 2xyy' = 0$.

Classify them into the following categories:

i. Separable Equations:----------

ii. First Order Linear Differential Equations:……..

iii. Exact Equations:……..

iv. Nonlinear Differential Equations:……..

Q.2 (10 points) Consider the initial value problem (IVP): $y' = \dfrac{2x}{1+2y}$, $y(1) = 0$.

a) Find the solution of the IVP in explicit form.

b) By looking at the explicit form of the solution, determine the interval in which the solution is defined.

Q.3 (10 points) Consider the equation: $\left[4\left(\dfrac{x^3}{y^2}\right) + \left(\dfrac{3}{y}\right)\right] + \left[3\left(\dfrac{x}{y^2}\right) + 2y\right]y' = 0.$

a) Find an integration factor of the form $\dfrac{N_x - M_y}{M}$.

b) Solve the differential equation.

II. Questions on Seconder Order differential Equations:

Q.3 Consider the differential equation $xy'' - (x+1)y' + y = 0.$

a) Given that $y_1 = e^x$ is a solution of the differential equation, use method of reduction of order to find a second solution.

b) Check the Wronskian of y_1 and y_2 and decide whether they form a fundamental set of solutions.

Q.4 (10 points) Consider the differential equation $xy'' - (x+1)y' + y = 0.$

c) Given that $y_1 = e^x$ is a solution of the differential equation, use method of reduction of order to find a second solution.

d) Check the Wronskian of y_1 and y_2 and decide whether they form a fundamental set of solutions.

Q.5 (8 points) Consider the differential equation: $y'' + 3y' = 2t^4 + t^2 e^{-3t} + 2\sin 3t.$ Write a form for the general solution y_g if the method of undetermined coefficients is to be used. Do not evaluate the constants.

Q.6 (12 points) Consider the initial value theorem (IVP) $\begin{cases} y'' + 2y' + 6y = 0, \\ y(0) = 2, \\ y'(0) = \alpha \geq 0. \end{cases}$

a) Find the solution of this problem.

b) Find α such that $y = 0$ when $t = 1.$

c) Determine the end behavior of the solution as $t \to \infty.$
