Prince Sultan University

Department of Mathematics

and

General Sciences

Math 225

Final Exam Term 161 January 10, 2017

Duration: 180 minutes

Name:

Student Number:

Instructions:

1. Show your all work in details.
2. Use the given spaces to answer each question.
3. Use pencil to write your answer.

Grading policy:

Questions	Q.1,2,3	Q.4,5,6	Q.7,8	Q.9,10	Q.11	Q.12	Total
Question Mark	4+5+5	3+4+5	3+10	6+9	12	14	80
Student Mark							

Grade out of 40

Good Luck

Q.1 (4 pts.) Solve the following linear equation: $y' + 2y = e^{\frac{t}{2}}$.

Q.2 (5 pts.) Find the inverse Laplace transform of $F(s) = \dfrac{4s+2}{s^2+2s+5}$.

Q.3 (5 pts.) Find an appropriate form for the particular solution y_p for the equation $y'' - 9y' + 14y = 3x^2 - 5\sin 2x + 7xe^{7x}$. Do not evaluate the constants.

Q.4 (3 pts.) Find the values of n and m for which the equation $(xy^n + x^2)dx + (x^2y^m + y^3)dy = 0$ is exact.

Q.5 (4 pts.) Find the solution of $y''' + 3y'' + 3y' + y = 0$.

Q.6 (5 pts.) Find the solution of the Euler equation: $x^2y'' - 3xy' + 4y = 0$.

Q.7 (3 pts.) Determine an interval in which the solution of $(16-x^2)y' + 2xy = 3x^2,\ y(-5) = 1$ is certain to exist.

Q.8 (10 pts.) Consider the equation: $2xy^3dx + (3x^2y^2 + x^2y^3 + 1)dy = 0$. Solve it by finding an integrating factor.

Q.9 (6 pts.) Express the solution of the IVP: $y'' + 4y = g(t), \quad y(0) = 1, \quad y'(0) = 1$ in terms of convolution integral.

Q.10 (9 pts.) Given that $y_1(x) = x^4$ is a solution of the equation $x^2 y'' - 7xy' + 16y = 0$. Use the method of reduction of order to find a second solution $y_2(x)$ for the equation.

Q.11 (12 pts.) Consider the equation $(1-x)y'' + y = 0$. Find a power series solution about the point $x_0 = 0$. Express the solution form by finding the first three terms in each of the two solutions.

Q.12 (14 pts.) Find the solution $u(x,y)$ of Laplace equation $u_{xx}+u_{yy}=0$ that satisfies the boundary conditions $\begin{cases} u(0,y)=0, & u(2,y)=0, & 0<y<3 \\ u(x,0)=0, & u(x,3)=g(x) & 0\le x\le 2 \end{cases}$ where $g(x)=\begin{cases} x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2 \end{cases}$.

Follow these steps to provide your answer:

1. Use separation of variables to convert PDE to ODE's.
2. Solve the ODE's.
3. Use Fourier series to find the unknown coefficients.