Prince Sultan University

Department of Math. & General Sciences

MATH 225

Final Examination

Spring 2014-2015, Term 142

21 MAY, 2015

Time Allowed: 120 minutes

Student Name: _____

Student ID #: _____

Rules governing the exam:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. You must show all your work beside the problem. Be organized.
6. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
7. This examination has 9 problems. Make sure your paper has all these problems.

Grading:

Problems	1	2	3	4	5	6	7	8	9	Total
Maximum Points	8	6	6	8	10	10	10	10	12	80
Student's Points										

Q.1 (7 points) Find the general solution of the differential equation $y' - 4y = t^2 e^{4t}$ and use it to determine the behavior of solution as $t \to \infty$. .

Q.2 (7 points) Find the general solution of the differential equation $x^2 y'' + xy' - y = 0.$

Q.3 (8 points) Use table of Laplace transforms to find the Laplace transform of $f(t) = te^{-t} - \cos^2 t$.

Hint: Use the fact that $\cos^2 t = \dfrac{1+\cos 2t}{2}$.

Q.4 (6 points) Find the general solution of $y''' + 3y'' - y' - 3y = 0$.

Q.5 (10 points) Consider the differential equation $ydx + (2xy - e^{-2y})dy = 0.$

a) Show that the differential equation is not exact.

b) Find an integrating factor which makes the equation exact.

c) Solve the differential equation.

Q.6 (10 points) Consider the differential equation $2xy'' + 2y' - y = 0$.

a) Show that the differential equation has a regular singular point at $x = 0$.

b) Determine the indicial equation, the roots of the indicial equation and the recurrence equation.

c) Find a series solution ($x > 0$) corresponding to one root.

Q.7 (10 points) Express the solution of the initial value problem $\begin{cases} y'' + 2y' + 2y = \sin \alpha t \\ y(0), = 0, y'(0) = 1 \end{cases}$ in terms of a convolution integral.

Q.8 (10 points) Given that $y_1 = e^t$ is a solution of the differential equation $ty'' - (t+1)y' + y = 0,$ $t > 0,$ find the general solution.

Q.9 (12 points) Find the general solution (expressed in terms of real valued functions) of the following system $\vec{x}'(t) = \begin{bmatrix} -1 & 5 \\ -2 & -3 \end{bmatrix} \vec{x}(t)$.