



Prince Sultan University  
Department of Mathematics and Physical Sciences

Math 223  
First Midterm Examination  
Semester II, Term 112  
Sunday, April 1, 2012

Time Allowed: 90 minutes

Name:

Student Number:

**Important Instructions**

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 7 problems, some with several parts. Make sure your paper has all these problems.

**Question.1 (8 points)**

Suppose that the augmented matrix for a system of linear equations has been reduced by

row operations to the reduced row echelon form  $\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right]$ . Solve the system.

Question.2 (11 points)

Let  $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$ . Compute the following:

- a)  $\frac{1}{2}C^T B - \frac{1}{4}A$ .
- b)  $\text{tr}(A)$
- c)  $B^3$

Question.3 (12 points)

- a) Given  $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$ . Find  $\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$ .
- b) Let  $\det(A) = -7$ . Find  $\det[(2A)^{-1}]$ .
- c) Find  $A$  if  $(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$ .

Question.4 (10 points)

- a) Determine whether  $\mathbf{u} = (6, 1, 4)$  and  $\mathbf{v} = (2, 0, -3)$  make an acute, obtuse or right angle.
- b) Let  $\mathbf{v} = (-1, 2, 5)$ . Find all scalars  $k$  such that  $\|k\mathbf{v}\| = 4$ .

Question.5 (12 points)

- a) Find the area of the triangle having vertices  $P = (1, -2, 2)$ ,  $Q = (0, 3, 4)$ ,  $R = (6, 1, 8)$ .
- b) Determine whether  $\mathbf{u} = (5, -2, 1)$ ,  $\mathbf{v} = (4, -1, 1)$ ,  $\mathbf{w} = (1, -1, 0)$  lie in the same plane when positioned so that their initial points coincide.

Question.6 (12 points)

- a) Find parametric equations for the line of intersection of the planes  $7x - 2y + 3z = -2$  and  $-3x + y + 2z = -5$ .
- b) Find the distance between the parallel planes  $3x - 4y + z = 1$  and  $6x - 8y + 2z = 3$ .

Question.7 (15 points)

Consider the system  $\begin{cases} x + 2y = \lambda x \\ 2x + y = \lambda y \end{cases}$ . Express the system in the form  $(\lambda I - A)x = 0$ . Then

- a) Find the characteristic equation
- b) Find the characteristic values
- c) Find the characteristic vectors corresponding to each characteristic values.