Prince Sultan University

Department of Mathematics and Physical Sciences

Math 223
Second Midterm Examination
Semester II, Term 112
Sunday, May 10, 2012

Time Allowed:90 minutes

Name:
Student Number:

Important Instructions

:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 5 problems, some with several parts. Make sure your paper has all these problems.

Question	1,2	3,4	5	Total/45	Total/22.5
Points					

Good Luck

Question.1 (8 points)

a) For which values of *k*, the vectors $u = (k, k, 1)$ and $v = (k, 5, 6)$ orthogonal.

b) Use the Wronskian to show that the set $\{1, e^x, xe^x\}$ is linearly independent.

Question.2 (9 points)

a) Find eigenvalues of A^{10} if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0.5 & 4 \\ 0 & 0 & 7 \end{bmatrix}$.

b) Determine whether or not the vectors $v_1 = (2, 2, 2)$, $v_2 = (0, 0, 3)$, $v_3 = (0, 1, 1)$ span R^3.

Question.3 (7 points) Consider the system $\begin{cases} 3x + y + z + w = 0 \\ 5x - y + z - w = 0 \end{cases}$.

a) Find a basis for the solution space of the system.

b) Determine the dimension of the solution space.

Question.4 (9 points)

a) Show that the linear operator $T : R^2 \to R^2$ defined by the equations $\begin{cases} w_1 = x + 2y \\ w_2 = -x + y \end{cases}$ is one to one.

b) If so, find the standard matrix for the inverse operator.

c) Find $T^{-1}(w_1, w_2)$.

Question.5 (12 points)

a) Find the eigenvalues and the corresponding eigenvectors of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

b) Find a matrix P that orthogonally diagonalizes A.

c) Determine $P^{-1}AP$.