Prince Sultan University
Department of Mathematics and Physical Sciences

Math 223
First Midterm Examination
Semester I, Term 101
Monday, November 8, 2010

Time Allowed: 100 minutes

Name:

Student Number:

Important Instructions

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1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 9 problems, some with several parts. Make sure your paper has all these problems.

Problems	**Max points**	**Student's Points**
1,2,3,4	22	
5,6	16	
7,8	18	
9	9	
Total	**75**	

Question.1 (5 points)

Assuming that all matrices are $n \times n$ and invertible, solve $ABC^T DBA^T C = AB^T$ for D.

Question.2 (5 points)

Let A be an $n \times n$ symmetric matrix. Show that A^2 is symmetric.

Question.3 (6 points)

Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ where b is a nonzero real number.

a) Find $\det(A)$.

b) Find $\det(A^{-1})$.

Question.4 (6 points)

Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the reduced row echelon form $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & 1 & 3 & 2 \end{array}\right]$. Solve the system.

Question.5 (6 points)

Let $A^{-1} = \begin{bmatrix} -2 & 7 & -1 \\ 1 & -4 & 1 \\ 4 & -13 & 2 \end{bmatrix}$ be the inverse of $A = \begin{bmatrix} 5 & -1 & 3 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$. Solve $\begin{cases} 5x - y + 3z = a \\ 2x + z = b \\ 3x + 2y + z = c \end{cases}$

where a, b, c are real numbers.

Question.6 (10 points)

Consider the matrices $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$.

a) Compute $tr(AC)$.

b) Compute $2B^T - D^{-2}$.

Question.7 (8 points)

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Assuming that $\det(A) = -7$. Find

a) $\det(A^{-1})$.

b) $\det\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$.

Question.8 (10 points)

Given the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 5 & 2 \\ -1 & 0 & 4 \end{bmatrix}$.

a) Find $\det(A)$ and $adj(A)$.

b) then use them to find A^{-1}.

Question.9 (9 points)

Show that the 4×3 linear system $\begin{cases} -x+y+z=9 \\ 2x+y-z=-10 \\ 3x-2z=-19 \\ -x+2y-3z=-10 \end{cases}$ has a unique solution.