Prince Sultan University

Department of Mathematics and Physical Sciences

Math 221
Final Examination
Semester II, Term 132
Thursday, May 22, 2014

Time Allowed: 120 minutes

Name: Student Number:

Statement of Ethics:

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

A deal: You are allowed to use the calculator but you have to show all details of your work.

Total/70:
Total/40:

Problem.1 (10 points) Perform four iterations of Newton's method for the polynomial $f(x) = 4x^3 - 2x^2 + 3$ starting with $x_0 = -1$.

Problem.2 (10 points) Consider the system $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix}.\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$. Apply the Gauss-Siedel iteration starting with $x^{(0)} = (0,0,0)^T$ to approximate the solution of the system. Proceed until you get $x^{(3)}$.

Problem.3 (15 points) Use Euler's method to approximate the solution of the initial value problem

$$\begin{cases} y' = te^{3t} - 2y, 0 \le t \le 1 \\ y(0) = 0 \end{cases}$$ with $h = 0.5$. Perform 5 iterations.

Problem.4 (15 points) Consider the integral $\int_{-2}^{2} x^3 e^x dx$.

a) Use scientific calculator to find the exact value of the integral.
b) Use the composite Trapezoidal rule with $n = 4$ to approximate the value of the integral.
c) Find a bound for the absolute error. Is the approximation obtained by CTR good enough?

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

Problem.5 (20 points) The table below lists values of a function at various points.

a) Use Lagrange interpolation to find a fourth order polynomial $p_4(x)$ that approximates these data. Find $p_4(1.5)$.
b) Construct the table of divided differences.
c) Use method of divided differences to find polynomial $p(x)$ that approximates the function $f(x)$.
d) Find $p(1.5)$ and compare it with the result obtained in part (a).