# Prince Sultan University

Department of Mathematics and Physical Sciences

Math 215
Final Examination
Semester I, Term 121
Sunday, January 13, 2013

Time Allowed: 120 minutes

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Name:

Student Number:

## Important Instructions

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There must be NO talking during the examination.
5. This examination has 10 problems, some with several parts. Make sure your paper has all these problems.

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## Grading Policy:

| Questions | Q.1 | Q.2 | Q.3 | Q.4 | Q.5 | Q.6 | Q.7 | Q.8 | Q.9 | Q.10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Student's Mark | | | | | | | | | | | |
| Maximum Mark | 5 | 5 | 6 | 8 | 6 | 9 | 8 | 9 | 9 | 15 | 80 |

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|---|
| 40 |

1. (5 points) Find the distance between the plane $2x-3y+6z=4$ and the point $(1,-4,-3)$.

2. (5 points) Describe the domain of the function $f(x,y)=\dfrac{\sqrt{4-x^2-y^2}}{\sqrt{x^2+y^2-1}}$.

3. (6 points) Find an equation of the plane through $(-2,1,7)$ that is perpendicular to the line $x-4=2t, y+2=3t, z=-5t$.

4. (8 points) Find $\iint_R \cos\sqrt{x^2+y^2}\,dA$ where $R$ is the region bounded by $x^2+y^2=4$.

5. (6 points) Let $f(x,y) = 5xe^{y} + 2x\sin xy$. Find $f_x - 5f_{yy}$.

6. (9 points) Given the matrix $A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$.

a) Find the eigenvalues of $A$.

b) Is $A$ diagonalizable? Explain your answer.

c) Find $A^5$.

7. (8 points) Find the surface area of the portion $z = x^2 + 2y$ between $y = x, y = 0$ and $x = 4$.

8. (9 points) Find the volume of the solid bounded by the graphs of $z = x^2$, $z = 1$, $y = 0$ and $y = 2$.

9. (9 points) Evaluate $\lim_{(x,y)\to(0,1)} \dfrac{x^2y - x^2}{x^2 + y^2 - 2y + 1}$

10. (15 points) Consider the matrix $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$.

a) Construct the characteristic equation and find the eigenvalues.
b) Find the eigenvectors corresponding to the eigenvalues.
c) Find a basis for the the eigenspace of $A$ and determine its dimension?
d) Find orthonormal basis for each eigenspace.
e) Construct a matrix $P$ that orthogonally diagonalizes A.