Prince Sultan University

MATH 111

Major Exam II

Semester I, Term 161

Monday, December 19, 2016

Time Allowed: **90 minutes**

Student Name: ____

Student ID #: ____

Teacher's Name: _________________________________Section #: __________

Serial #: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. You have to show your work in details in all problems.
8. This examination has 12 problems, some with several parts. Make sure your paper has all these problems.

Page #	Max points	Student's Points
1	16	
2,3,4,5	24	
6,7,8,9	20	
10,11,12	20	
Total	**80**	

Q1. (16 points) Compute y'. Simplify where possible. **(Note: Show your work in details)**

(a) $y = \dfrac{5}{\sqrt[3]{\tan x + 2^x}}$

(b) $y = \tan^{-1}(\sin^{-1}(\sqrt{x}))$

(c) $\sin(xy) = \dfrac{y}{x}$

(d) $y = (\cos x)^x$

(e) $y = \ln\left(\coth\left(\dfrac{1}{x}\right)\right)$

Q2. (5 points) Find an equation of the **normal** line to the curve $y=(2+5x)e^{-x}$ at the point $(0,2)$.

Q3. (6 points) At what point (x,y) on the curve $y=\left[\ln(x+4)\right]^2$ is the tangent horizontal?

Q4. (6 points) The volume of a cube is increasing at a rate of $8 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 24 cm? **(Note: Show your work in details)**

Q5. (7 points) A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2 \text{ cm}^3/\text{sec}$, how fast is the water level rising when the water is 5 cm deep? **(Note1: Volume of cone is:** $V=\frac{1}{3}\pi r^2 h$ **) (Note2: Show your work in details).**

Q6. (5 points) Suppose g is a differentiable function such that $g(x)+x\sin(g(x))=x^2$, find $g'(0)$.

Q7. (4 points) Find the limit: $\lim_{x\to 0}\dfrac{\tan 2x}{x^2-8x}$. **(Note: Show your work in details)**

Q8. (5 points) Find the critical numbers of the function: $f(x)=|2x+1|$. **(Note: Show your work in details)**

Q9. (6 points) (a) Prove the identity $\dfrac{1+\tanh x}{1-\tanh x}=e^{2x}$. **(Note: Show your work in details)**

(b) Show that $\dfrac{d}{dx}\sqrt[4]{\dfrac{1+\tanh x}{1-\tanh x}}=\dfrac{e^{x/2}}{2}$. **(Note: Show your work in details)**

Q10. (6 points) Find the absolute maximum and absolute minimum values of the function $f(x) = (x^2 - 1)^3$ on the interval $[-2, 2]$. **(Note: Show your work in details)**

Q11. (7 points) (a) Show that the equation $x^3 + e^{2x} = 0$ has a real zero between -1 and 0.

(b) Use Rolle's Theorem to show that the equation $x^3 + e^{2x} = 0$ has exactly one real root between -1 and 0.

Q12. (7 points) **Verify** that the function $f(x) = \dfrac{x}{x+1}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$. Then **find** all numbers c that satisfy the conclusion of the Mean Value Theorem.