

Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences



COURSE DETAILS:

Calculus I		MATH 111	Final EXAM
Semester:	Second Semester --Term 182		
Date:	Saturday April 20, 2019		
Time Allowed:	3 Hours		

STUDENT DETAILS:

Student Name			
Student ID #			
Section #		Attendance serial #	
Instructor's Name			

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Total	Total
Questions								
Marks	15	20	20	15	15	15	100	40

Q.1 (6 points) Let $f(x) = \begin{cases} 7 - 2x & \text{if } x \leq -2 \\ 3 & \text{if } -2 < x \leq 5 \\ 2x + 5 & \text{if } x > 5 \end{cases}$.

(a) Find the values of x , if any, at which f is not continuous. (**justify your answer**)

(b) Find the values of x , if any, at which f is not differentiable. (**justify your answer**)

(c) Sketch the graph of the first derivative of the above function

Q2 (4 points) Discuss the continuity of the function $f(x) = \frac{\sin x}{1 + \cos x}$.

Q.3 (5 points) Verify that the function $f(x) = \sqrt{25 - x^2}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-5, 3]$. Then find all numbers c that satisfy the conclusion of the Theorem.

Q.4 (9 points) Let $f(x) = \ln\left(\frac{x}{x-2}\right)$.

(a) Give the domain of f . (**Justify your answer**)

(b) Give the equations of the vertical and horizontal asymptotes of the graph of $f(x)$. (**Justify your answer**)

(c) Find the slope of the tangent line to the graph of the function $f(x) = \ln\left(\frac{x}{x-2}\right)$ at $x = -1$.

Q.5 (5 points) Find the critical numbers of the function: $f(x) = \sqrt[3]{9 - x^2}$.

Q.6 (6 points) Find the absolute minimum and the absolute maximum values of the function $f(x) = x\sqrt{x+1}$ over the interval $[-1, 1]$.

Q.7 (8 points) let $f(x) = x^3 - 3x + 5$

a. Find the intervals on which f is increasing or decreasing. (**Justify your answer**)

b. Find the intervals on which f is concave upward or downward. (**Justify your answer**)

Q.8 (5 points) Find two positive numbers whose product is 49 and whose sum is minimum.

Q.9 (7 points) Find the points on the hyperbola $x^2 - y^2 = 1$ that are closest to the point $(0, -3)$.

Q.10 (15 points) Calculate y'

(Note: simplify your answer as much as possible)

(a) $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

(b) $y = \frac{\sqrt{1-x^2}}{x}$

(c) $y = \ln(\ln x - \sin^{-1} x)$

(d) $y = (\tanh x)^{\sqrt[3]{x}}$

(e) $\cos(xy) = \sec(x^2 + y^2)$

Q.11 (15 points) Find the limit

Show your work in details

(a) $\lim_{x \rightarrow -\infty} (2 + e^x)^{-2}$

(b) $\lim_{x \rightarrow 0^+} \frac{\tan^{-1} x}{3x}$

(c) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)^x$

(d) $\lim_{x \rightarrow 0} \left(\frac{\sinh x - x}{x^3} \right)$

(e) $\lim_{x \rightarrow 0^+} \left(\frac{3}{x} - \frac{1}{e^x - 1} \right)$

Q.12 (15 points) Let $f(x) = \frac{x}{x^2 - 4}$.

Show your work in details

(a) (2 points) Find the domain of f and the y -intercept.

(b) (2 points) Determine the vertical and horizontal asymptotes, if any.

(c) (4 points) Find the critical numbers and the local maximum and/or local minimum points, if any.

(d) (4 points) Find the intervals on which f are concave up and/or concave down and the inflection point(s) if any.

(e) (3 points) Sketch the graph of f