Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

COURSE DETAILS:

Calculus I	MATH 111	Final EXAM
Semester:	First Semester --Term 181	
Date:	Saturday December 15, 2018	
Time Allowed:	3 Hours	

STUDENT DETAILS:

Student Name			
Student ID #			
Section #		Attendance serial #	
Instructor's Name			

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Page 5	Total	Total
Questions							
Marks	**16**	**16**	**20**	**14**	**14**	**80**	**40**

Q.1 (2 points) Discuss the continuity of the function: $g(x) = \dfrac{\sqrt[3]{x^2 - 9}}{5 - x^2}$

Q.3 (4 points) Find the points on the graph of $y^3 - x = 0$ where the tangent line has slope $\dfrac{1}{3}$.

Q.4 (5 points) Determine the intervals in which the function $f(x) = (x^2 - 1)^{10}$ is increasing or decreasing.

Q.5 (5 points) Find the absolute maximum and absolute minimum values of the function $f(x) = x^4 e^{-x}$ on the interval $[-1, 5]$.

Q.6 (16 points) Find the limit **Show your work in details**

(a) (3 points) $\lim_{x\to 1}\dfrac{1-x+\ln x}{1+\cos(\pi x)}$

(b) (3 points) $\lim_{x\to\infty}\dfrac{e^{2x}-e^{-2x}}{\ln(x+1)}$

(c) (3 points) $\lim_{x\to 0^+}(\dfrac{1}{x}-\dfrac{1}{x^3})$

(d) (3 points) $\lim_{x\to 0^-} xe^{-1/x}$

(e) (4 points) $\lim_{x\to 0^+}(1+\sin 3x)^{1/x}$

Q.7 (12 points) Calculate y' **(Note: Do not simplify your answer)**

(a) (3 points) $y = \sqrt[3]{\csc(11x)} + \ln 7$

(b) (3 points) $y = \ln(4^{\sin x} + \sin^{-1}(5x^3))$

(c) (3 points) $y = \dfrac{\sinh(7x)}{(5x^2+9)}$

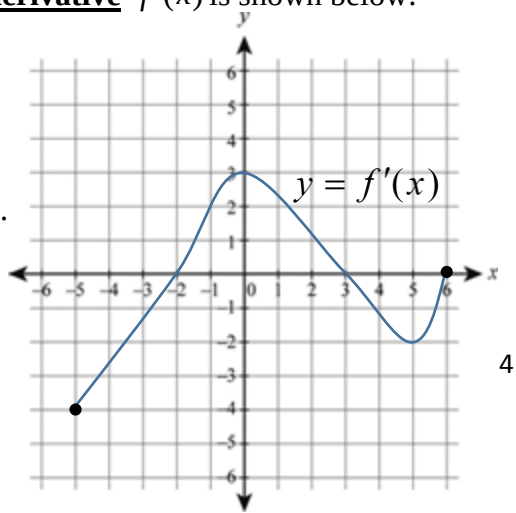
(d) (3 points) $\ln(8y^2) = 12 + y^3 e^{-2x}$

Q.8 (4 points) If $y = \ln\left(\dfrac{e^{-3}}{e^{2x}+e^{-2x}}\right)$ show that $y' = -2\tanh(2x)$

Q9 (4 points) Consider a function $f(x)$ for which the graph of **its derivative** $f'(x)$ is shown below:

a. (2 points) Find intervals in which $f(x)$ is increasing.

b. (2 points) Find the interval(s) in which $f(x)$ is concave upward.



Q.10 (14 points) Let $f(x) = e^{2x} - e^{x}$ **Show your work in details**

(a) (2 points) Find the domain of f and the intercepts (if any).

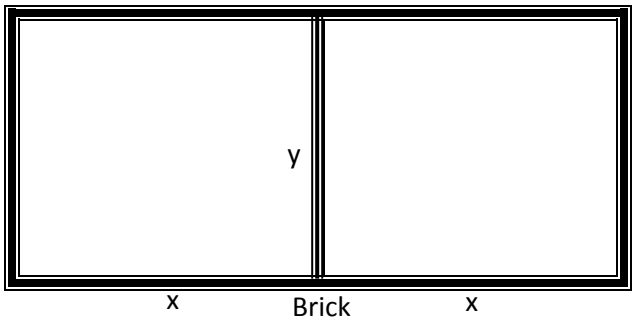
(b) (2 points) Determine the vertical and horizontal asymptotes (if any).

(c) (3 points) Find the critical numbers and the local maximum and/or local minimum points, if any.

(d) (4 points) Find the intervals on which f are concave up and/or concave down and the inflection point(s) if any.

(e) (3 points) Sketch the graph of f showing on the graph all significant features

Q.11 **(**6 points) A total area of 2400 m^2 is to be enclosed in two pens, as illustrated. The walls are to be constructed of brick, and the inner wall divides the field into two equal parts. Find the dimensions x and y that minimize the cost of construction.



Q.12 (4 points) A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.5 m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 m from the wall?

Q.13 **(**4 points) Let $f(x)=\frac{(x-1)^3}{x^2}$ and $f'(x)=1-\frac{3}{x^2}+\frac{2}{x^3}$ be given, find the coordinate of inflection point(s), (if any).