Prince Sultan University

Department of Mathematics and Physical Sciences

Math 223
Final Examination
Semester II, Term 112
Thursday, May 17, 2012

Time Allowed:120 minutes

Name:

Student Number:

Important Instructions

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 12 problems, some with several parts. Make sure your paper has all these problems.

Questions	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10	Q.11	Q.12	Total
Student Marks													
Maximum Marks	4	4	4	4	6	7	7	6	9	7	9	14	80

Question.1 (4 points) Find the eigenvalues of A^{20} for $A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & 0.5 & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

Question.2 (4 points) Find the scalar triple product $v_1 \cdot (v_2 \times v_3)$ if $v_1 = (-1,2,4)$, $v_2 = (3,4,-2)$ and $v_3 = (-1,2,5)$.

Question.3 (4 points) Let $v_1 = (2,6,-7)$ and $v_2 = (-1,-1,8)$. If $(2,14,11) = 3v_1 + lv_2$, what is the value of l?

Question.4 (4 points) Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & i & h \end{vmatrix} = -6$, find $\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4i & 4h \end{vmatrix}$.

Question.5 (6 points) Let $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$. Verify that $(B^T)^{-1} = (B^{-1})^T$.

Question.6 (7 points) Find the coordinate vector of $v=(2,-1,3)$ relative to the basis $S=\{v_1,v_2,v_3\}$ where $v_1=(1,0,0)$, $v_2=(2,2,0)$ and $v_3=(3,3,3)$.

Question.7 (7 points) Find an equation of the plane passing through the points $P(-4,-1,-1)$, $Q(-2,0,1)$ and $R(-1,-2,-3)$.

Question.8 (6 points) Consider the system $\begin{cases} 4x+5y=2 \\ x+5y+2z=1 \\ 11x+y+2z=3 \end{cases}$. Use Cramer's rule to find **the value of *z*.**

Question.9 (9 points) Consider the basis $S=\{v_1,v_2,v_3\}$ for R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0)$. Let $T:R^3\rightarrow R^3$ be the linear transformation such that $T(v_1)=(2,-1,4)$, $T(v_2)=(3,0,1)$ and $T(v_3)=(-1,5,1)$.

a) Find a formula for $T(x_1,x_2,x_3)$.

b) Evaluate $T(2,4,-1)$.

Question.10 (7 points) Let $T: R^3 \to R^3$ be multiplication by $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$. Determine whether T has an inverse. If so, find $T^{-1}(x_1, x_2, x_3)$.

Question.11 (9 points) Determine the dimension of and a basis for the solution space of the system

$$\begin{cases} 3x + y + z + w = 0 \\ 5x - y + z - w = 0 \end{cases}.$$

Question.12 (14 points) Let $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

a) Find the eigenvalues of A.

b) Is A diagonalizable? Justify your answer.

c) If A is diagonalizable, find a matrix P that diagonalizes A.

d) Determine $P^{-1}AP$.