



Prince Sultan University
Department of Mathematical Sciences
MATH 221 – Major Examination
02 January 2008

Time allowed: 120 minutes

Maximum points: 100 points

I.D #: _____ **Name:** _____ **Sign.:** _____

Q1 [15 points]: Use the *bisection method* to find *solutions* accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 16 = 0$ on each interval.

(a). $[0, 1]$

(b). $[3.2, 4]$

Q2 [12 points]: Apply the *Newton-Raphson's method* to approximate a root of the equation $x^3 + 4x^2 - 10 = 0$ correct to the eight decimal places; use initial approximation $p_0 = 1.5$.

Q3 [14 points]: Let $f(x) = -x^3 - \cos x$. Take $p_0 = -1$ and $p_1 = 0$. Find p_3 using
(a). *Secant method* (b). *Method of false position*

Q4 [15 points]: Let $f(x) = \cos x$. Take $x_0 = 0$, $x_1 = 0.6$ and $x_2 = 0.9$. **Construct** the *Lagrange interpolating polynomials of degree at the most **one*** and *at the most **two*** to **approximate** $f(0.45)$; also **find** the *absolute errors*.

Q5 [15 points]: Develop the divided-difference table to compute the fourth divided difference corresponding to the following data:

x	1	1.3	1.6	1.9	2.2
$f(x)$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

Then construct the Lagrange interpolation polynomial $\mathbf{P_4(x)}$ to approximate $f(1.5)$.

Q6 [15 points]: (a). **State** the following formulas to approximate $f'(x_0)$: *Forward-difference formula; Backward-difference formula; Three-point formulas; Five-point formulas.*

(b). **Apply** appropriate five-point formula with **h= 0.1** to **approximate $f'(2.0)$** for which the data is provided as below:

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.889365	12.703199	14.778112	17.148957	19.855030

Q7 [14 points]: **State** the *Trapezoidal* and *Simpson'* rules for *numerical integration*. Give a comparison between the two rules by approximating

$$\int_0^2 f(x) dx \quad \text{for } f(x) = x^4, x^{-1}, \sin x, e^x.$$

(Take care of yourself!)