Prince Sultan University

Department of Mathematical Sciences

MATH 221 – Major Exam 1

03 November 2007

Time allowed: 120 minutes **Maximum points: 100 points**

Q1 [**12** points]**:**

i. **Find** the *second* and *third Taylor polynomials* $P_2(x)$ and $P_3(x)$ for $f(x) = \cos x$ about $x_0 = 0$.

ii. **Approximate** $\cos 0.01$ using $P_2(x)$ and $P_3(x)$.

iii. **Compare** the above two *approximation techniques*.

Q2 [**14** points]**:**

Find the *decimal number* represented in the *binary floating-point system* as **0 10000000011 1011100100010000000000000000000000000000000000000000**. Also **find** its *next smallest* and *next largest machine numbers*.

Q3 [**12** points]**:**

i. **Give** a *comparision* between *absolute error* and *relative error* for an *approximation* p^* to p.

ii. **If** $fl(y) = 0.d_1d_2..d_kd_{k+1}\times 10^n$ is k*-digit decimal floating-point* representation of the number $y = 0.d_1d_2d_3\ldots\times 10^n$ by using the *chopping,* **then show** that the *relative error* is $\leq 10^{-k+1}$.

Q4 [**18** points]**:**

i. **Use** the *Intermediate Value Theorem* to **confirm** the *existence* of a *root* of the *equation* $x^3+4x^2-10 = 0$ in the *interval* **[1,2]**.

ii. **Apply** the *Bisection Algorithm* to **find an approximation** of the existing *root correct to at least four significant digits*.

iii. **Justify** the claim that *the Bisection Algorithim converges* with the *rate* $O(1/2^n)$.

Q5 [**14** points]**:**

i. **Using** the *pseudocode* **write** an *algorithm* for the *Newton-Raphson's method* to *find* an *approximate solution* of $f(x) = 0$ with given *initial approximation* p_0.

ii. **Apply** the above method **to find** a *zero* of the *function* $f(x) = \cos x - x$ in the *interval* $[0,\pi/2]$ *accurate* to *ten decimal places*; **use** the *initial approximation* $p_0=\pi/4$.

Q6 [**15** points]**:**

i. **Apply** the *Secant method* **to find** a *solution* of $\cos x - x = 0$, **using** *the initial approximations* as $p_0=0.5$ and $p_1=\pi/4$.

ii. **Apply** the *method of False Position* **to find** a *solution* of $\cos x - x = 0$, **using** the *initial approximations* p_0 and p_1 same as in Part i.

iii. **Compare** the *method of False Position* with the *Secant method.*

Q7 [**15** points]**:**

i. **Define** *linearly convergent* and *quadratically convergent sequences.*

ii. **Give an example to verify** that a *quadratically convergent sequence converges* more *rapidly* than a *linearly convergent sequence*.

iii. **Show** that the *Newton-Raphson's method converges quadratically*.

(Take care of yourself!)