Prince Sultan University

Department of Mathematics

Math 221-Numerical Analysis

Course Section: 071227

Final Examination

Semester I, Fall (071)

Sunday, January 27, 2008

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Time Allowed: **150** ***minutes*** ***Maximum Points:*** **100**

Student Name: ___________________________ Student ID #: __________________

Important Instructions:

- *You may use scientific calculator with programming or graphing capabilities.*
- *You may NOT borrow calculator from other candidate.*
- *Do NOT use red ink for writing your answers.*
- *You may NOT use notes or any textbook.*
- *There should be NO talking during the examination.*
- *Your exam paper will be taken immediately if your mobile phone is seen or heard.*
- *Looking around or making an attempt of cheating may cause you expulsion from the examination.*
- *This examination paper contains* **9** *questions (&* **10** *pages). Make sure you have received the paper containing all questions.*

Question #	Max. Points	Student's Points
1	10	
2	12	
3	10	
4	12	
5	10	
6	12	
7	12	
8	12	
9	10	
Total →	100	

Question 1 [**10** points]**:** Suppose that $x = 0.714251$ and $y = 98765.9$ and that five-digit chopping is used for arithmetic calculations involving x and y. *Perform the following computer-type operations* on the floating-point representations $fl(x) = 0.71425\times10^0$ and $fl(y) = 0.98765\times10^5$: $x \oplus y$, $x \otimes y$, $x \Theta y$, $x \oplus y$. Then compare the involved *absolute* and *relative errors.*

Question 2 [**12** points]**:** Find an approximate value of $\sqrt{3}$ correct to within 10^{-4} by using the following methods and then compare the two methods:

(i). Bisection method (ii). Secant method with $p_o=1$ and $p_1=2$.

Question 3 [**10** points]**:** A root of equation $\boldsymbol{x}^3+4\boldsymbol{x}^2-10 = 0$ exists in the interval [1,2]. Apply the *fixed-point iteration technique* to approximate a root of the equation $\boldsymbol{x}^3+4\boldsymbol{x}^2-10 = 0$ correct to the eight decimal places; use Newton-Raphson's technique to formulate corresponding fixed-point problem and take initial approximation $\boldsymbol{p}_o = 1.5$.

Question 4 [**12** points]**:** Make a table of divided differences and construct the 3rd Lagrange interpolating polynomial for the following data and use the polynomial to approximate $f(8.4)$:

$$f(8.1)=16.94410,\ f(8.3)=17.56492,\ f(8.6)=18.50515,\ f(8.7)=18.82091\ .$$

Question 5 [**10** points]**:** Apply the appropriate *five-point formula* with $\boldsymbol{h}= 0.1$ to approximate $\boldsymbol{f}'(2.0)$ by using the following data for function $\boldsymbol{f}(x) = xe^x$. **:**
$\boldsymbol{f}(1.8)=10.889365$, $\boldsymbol{f}(1.9)=12.703199$, $\boldsymbol{f}(2.0)=14.778112$, $\boldsymbol{f}(2.1)=17.148957$, $\boldsymbol{f}(2.2)=19.855030$

Question 6 [**12** points]**:** Approximate the integral $\int_0^1 x^2 e^{-x^2} dx$ using

(i). Composite trapezoidal rule with h = 0.25;
(ii). Simpson's rule with h = 0.5.

Question 7 [**12** points]**:** Show that the equation $y\sin t + t^2 e^y + 2y = 1$ implicitly defines a solution for the initial-value problem:

$$y' = -\left(\frac{y\cos t + 2te^y}{\sin t + t^2 e^y + 2}\right), \quad 1 \le t \le 2, \quad y(1) = 0.$$

Then approximate $y(2)$ using Newton-Raphson's method with $p_0 = 0$.

Question 8 [**12** points]**:** The actual solution to the initial-value problem:

$$y' = 1 + \frac{y}{t}\ ,\ 1 \le t \le 2\ ,\ y\left(1\right) = 2$$

is $y(t) = t\ln t + 2t$. Apply Euler's method with $h = 0.25$ to approximate the solutions of the above *IVP*. Then use the data so generated in the 3rd Lagrange interpolation polynomial to approximate y (1.6).

Question 9 [**10** points]**:** Use the Jacobi method with at least three iterations to find an approximate solution of the following linear system; initialize the algorithm from $X^{(0)} = (0,0,0)$:

$$\begin{aligned} 10x_1 - x_2 \qquad\quad &= 9 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6 \end{aligned}$$