Prince Sultan University
Department of Mathematical Sciences
MATH 221 – Major Exam 2
14 November 2007

Time allowed: 120 minutes **Maximum points: 100 points**

I.D #:_____ Name: _____ Sign.:_____

Q1 [**12** points]**:** Find the *third Taylor polynomial* $P_3(x)$ for $f(x) = e^x \cos x$ about $x_0 = 0$. Use this polynomial to approximate $f(0.5)$ and also compute the error/s involved.

Q2 [**12** points]**:** Represent the decimal number 27.56640625 in the *binary floating-point system.* Also find its *next smallest* and *next largest machine numbers*.

Q3 [**12** points]**:** Suppose that $x = 5/7$ and $y = 1/3$ and that five-digit chopping is used for arithmetic calculations involving x and y. *Perform the following computer-type operations* on the floationg-point representations $fl(x) = 0.71428\times10^0$ and $fl(y) = 0.33333\times10^0$: $x \oplus y$, $x \otimes y$, $x \ominus y$, $x \oplus y$. Also find *absolute* and *relative errors.*

Q4 [**14** points]**:** A root of the equation $x^3+4x^2-10 = 0$ correct to nine decimal places is 1.365230013. Use the *Bisection Method* to find an approximate *root* of the *equation* in the *interval* [1,2] correct to at least four significant digits. Also discuss convergence of this method.

Q5 [**12** points]**:** Explain the fact that every fixed-point problem corresponds to a root-finding problem and vice a versa. Apply the *fixed-point iteration* technique on $g(x)= x - (x^3+4x^2-10)/(3x^2+8x)$ to approximate a root of the equation $\boldsymbol{x^3+4x^2-10 = 0}$ correct to the eight decimal places; take initial approximation $p_0 = 1.5$.

Q6 [**12** points]**:** Apply the *Newton-Raphson's method* to find a *zero* of the *function* $\boldsymbol{f(x) = \cos x - x}$ in the *interva*l [0,π/2**]** *accurate* to *ten decimal places*; use the *initial approximation* $\boldsymbol{p_0}=\pi/4$.

Q7 [**14** points]**:** Apply the *Secant method* to find a *solution* of $\cos x - x = 0$, using *the initial approximations* as $p_0=0.5$ and $p_1=\pi/4$. Also give a comparison between *secant method* and the *Newton-Raphson's method.*

Q8 [**12** points]**:** Let $P_3(\mathbf{x})$ denote the *third Lagrange interpolating polynomial* for the data (0,0), (0.5, **y**), (1,3), and (2,2). Find the value of **y** if the coefficient of $\mathbf{x}^3$ in $P_3(\mathbf{x})$ is **6**.

(Take care of yourself!)