1) Consider the function $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

   a) Show that the function f has the critical points $(0,0)$, $(0,4)$,(2,2), (-2,2).

   b) Find $f_{xx}(x, y),\ f_{xy}(x, y),\ and\ f_{yy}(x, y)$.

   c) Use the second derivative test to find the local maximum and minimum values and saddle points of f if any exist.

2) a) Evaluate the integrals:

i) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$ ii) $\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$.

b) Find the volume of the solid enclosed by the cone $z = \sqrt{x^2+y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$. (Hint: integrate over the region enclosed by the circle of intersection).

3) Consider the set $S = \{v_1,\ v_2, v_3\}, v_1 = (1,0,0),\ v_2 = (3,7,-2),\ v_3 = (0,4,1)$ of vectors or R^3.

a) Show that S is a basis for R^3.

b) Use Gram-Schmidt process to transform the basis S into an orthonormal basis $T = \{u_1, u_2, u_3\}$.

c) Find the coordinate vector $(w)_T$, where $w = (1,1,1)$.

4)a) Find a matrix P that orthogonally diagonlizes $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Then, determine $P^{-1}AP$ and calculate the matrix A^{10}.

b) Prove that if $\mu \neq 0$ is an eigenvalue of an invertible matrix A with eigenvector X then μ^{-1} is an eigenvalue of A^{-1} with eigenvector X.

c)Prove that if A is invertible and symmetric then A^{-1} is symmetric as well.

d) Find the eigenvalues and eigenvectors of the matrix A^{-1} where $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Then, find a matrix P , if possible, that orthogonally diagolaizes A^{-1} and determine $P^{-1}A^{-1}P$.