Math 215 Engineering Mathematics

Major 1 Fall 2014-2015

November 14, 2014

20 marks for each question

1)a) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2+sin^2y}{2x^2+y^2}$ does not exist.

b) Show that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \texttt{if } (x,y) \neq (0,0) \\ 0 & \texttt{if } (x,y) = (0,0) \end{cases}$ is continuous at $(0,0)$, by using the definition.

2) a) If $z = f(x-y)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

b) If $w = xy + yz + zx$, $x = rcost$, $y = rsint$ and $z = rt$, find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial t}$ at the point $(r,t) = (2, \pi/2)$.

3) For the function $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$, find the critical points and classify them.

4)a) Evaluate the double integral $\int\int_R xsin(x+y)dA$, where $R = [0, \pi/6] \times [0, \pi/3]$.

b) Evaluate the double integral $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} cos(x^2)dxdy$.

5) a)Evaluate the integral $\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} sin(x^2+y^2)dydx$ by converting to polar coordinates.

b) Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.