Prince Sultan University

Math 211
Final Exam
Semester 171
Saturday, January 6, 2018
Time Allowed: 3 hours

Student Name:_____

Student ID #: _____

Important Instructions:

1.You may use a scientific calculator that does not have programming or graphing capabilities.
2.You may NOT borrow a calculator from anyone.
3.You may NOT use notes or any textbook.
4.Talking during the examination is NOT allowed.
5.Your exam will be taken immediately if your mobile phone is seen or heard.
6.Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1-5	22	
6-8	18	
9-11	16	
12	14	
13-15	16	
16-18	14	
Total	**100**	
Total	**40**	

1) [4 pts] Find the domain of $f(x)=\dfrac{\sqrt{x^2-x-6}}{x-3}$

2) [4 pts] When x units of a commodity are produced, the total cost will be $C(x)=0.5x^2+3x+12$ dollars, then all x units will be sold at a price of $p(x)=9-0.2x$ dollars per unit. Find the **marginal profit** of producing and selling the 11^{th} unit.

3) [6 pts] When the price of a certain commodity is p dollars per unit, consumers demand x hundred units of the commodity, where $77x^2+19p^2=5300$. How **fast is the demand x changing** with respect to time when the price is \$8 and is decreasing at a rate of 75 cents per month?

4) [4 pts] Use **implicit differentiation** to find y' the first derivative of $x^3y^3-y=x$

5) [4 pts] Find the x value where the graph of $f(x)=\dfrac{x^2}{x-3}$ has a horizontal tangent line.

6) [6 pts] Find the absolute maximum and minimum of $f(t) = 3t^5 - 5t^3$ over the interval $-2 \le t \le 2$

7) [6 pts] Given the function $f(x) = x(x-4)^3$. Find the intervals of concave up and concave down. Find the coordinated of the inflection points, if any.

8) [6 pts] Find the first derivative of the following:

a) $y = 3^{(3x^2+2x)}$

b) $y = \log_4 \sqrt[3]{e^x + \frac{1}{x}}$

c) $y = \frac{x^2(x-1)}{\sqrt{x+2}}$ (use logarithmic differentiation)

9) [6 pts] When a commodity is sold for *p* dollars per unit customers will buy $D(p) = \frac{40,000}{p}$ units per month. It is determined that *t* months from now, the price will be $p(t) = 0.4t^{3/2} + 6.8$ dollars per unit. At what rate will the monthly demand be changing with respect to time 4 months from now?

10) [6 pts] Find the following integrals:

a) $\int \frac{8x-2}{6x^2-3x+1}\,dx$

b) $\int \left(\frac{1}{x(\ln x)^2} \right) dx$

11) [4 pts] It is estimated that **<u>*x* years from now</u>** the value of acre of land $V(x)$ will be increasing at the rate $V'(x) = \frac{0.4x^3}{\sqrt{0.2x^4 + 8000}}$ dollars per year. If the land is **<u>currently</u>** worth \$500/acre, find the Value function $V(x)$.

12) [2+4+4+4 pts] Evaluate the following definite integrals:

a) $\int_2^5 \left(2+2t+3t^2\right)\,dt$

b) $\int_0^1 \left(x^3+x\right)\sqrt{x^4+2x^2+1}\,dx$

c) $\int_2^{e+1} \left(\frac{x}{x-1}\right)dx$

d) $\int_1^4 \frac{\left(\sqrt{x}-1\right)^{3/2}}{\sqrt{x}}\,dx$

13) [6 pts] A broker's investment portfolio changes value at the rate $V'(t) = 12e^{-0.05t}\left(e^{0.3t} - 3\right)$, where V is in thousands of dollars and t is the number of years **<u>after</u>** 2006. What was the **<u>net change</u>** in the value of the broker's portfolio between 2010 and 2012?

14) [6 pts] Find the area of the regions bounded by the curves $y = x^3 + x^2 + 5x$ and $y = 2x^2 + 11x$

15) [4 pts] Find the vertical and horizontal asymptotes of the graph of $f(x) = \dfrac{2x^2}{3x^2 - 9}$

16) [2 pts] Find the domain of $f(x,y)=\dfrac{\sqrt{x}}{\ln(x-y)}$

17) [6 pts] Suppose that at a certain factory, output is given by the *Cobb-Douglas* production function $Q(K,L)=20K^{0.15}L^{0.85}$ units, where K is the capital investment measured in units of \$1,000 and L the size of the labor force measured in worker-hours.

a) Compute the output if the capital investment is \$56,500 and the size labor force is 600 worker hours.

b) Find the **marginal productivity of capital** when the capital investment is \$120,000 and the size of the labor force is 800 worker hours.

c) Find the **marginal productivity of labor** when the capital investment is \$120,000 and the size of the labor force is 800 worker hours.

d) Should the manufacturer consider adding a unit of capital or a unit of labor to increase output more rapidly? Explain your answer.

18) [6 pts] Given the function $f(x,y)=5x^4y^3+2xy$ find the following partial derivatives:

a) f_x =

b) f_{xy} =

c) f_{xx} =