Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

Calculus I	MATH 111	MAJOR EXAM I
Semester:	Spring Semester --Term 182	
Date:	Sunday, February 17, 2019	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:			
Student ID Number:			
Section #:		Attendance Serial #:	
Instructor's Name:			

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Total	Total
Questions						
Marks	**16**	**12**	**17**	**15**	**60**	**20**

Q1. [7 pts] Let $f(x) = -1 + \ln(-x+2)$.

a) (2 point) Find the domain of $f(x)$

b) (5 points) Find the inverse of $f(x)$. Sketch the graph of $f^{-1}(x)$ and its asymptote.

Q2. [9 pts] Find the domain of the following functions:

a) $f(x) = \dfrac{\sqrt{e^x - 4}}{e^x + 4}$

b) $g(x) = \dfrac{\sqrt{x^2 - 4}}{x+3}$

c) $h(x) = \ln\left(\dfrac{2x}{x+2}\right)$

Q3. [4 pts] Let $f(x)=\frac{3}{x}$ and $g(x)=\frac{x+1}{x-2}$ find $g \circ f(x)$ **and** it's domain.

Q4. [2 pts] The following limit $\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h}$ represents the derivative of some function f at some number a. State the function f and the number a.

Q5. [2 pts] If an equation of the tangent line to the curve $y=f(x)$ at the point where $a=5$ is $y=7x-15$, find $f(5)$ and $f'(5)$.

Q6. [4 pts] Find the values of the constants a and b that makes the function f defined by $f(x)=\begin{cases} ax-4 & \text{if } x\le -1 \\ b+x^3+1 & \text{if } -1<x\le 1 \\ \frac{a(x^2-1)}{x-1} & \text{if } x>1 \end{cases}$, is continuous everywhere.

Q7. [2 pts] If $\lim_{x\to 1}\dfrac{f(x)-5x+3}{x-1}=17$, find $\lim_{x\to 1} f(x)$.

Q8. [15 pts] Find each of the following limits, if it exists: **[Note: Do not use L'Hospital's Rule]**

i. $\lim_{x\to\infty}\dfrac{\sin^3 x}{x^4+1}$

ii. $\lim_{x\to 2^-}\dfrac{x^3-2x^2}{|x-2|}$

iii. $\lim_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x-2}$

iv. $\lim_{x\to 4^-}\dfrac{\sqrt{x}}{(x-4)^3}$

v. $\lim_{x\to -1}\dfrac{x^2+2x+1}{x^4-1}$

Q9. [2 pts] Use the definition of continuity and properties of limits to show that the function $f(x)=\frac{1-2x}{3+x}$ is continuous on the interval $(-\infty,-5)$.

Q10. [4 pts] Let $f(x)=\frac{1-2x}{3+x}$. Use the definition of the derivative to find $f'(0)$.

Q11. [9 pts] Find the horizontal and vertical asymptotes of each curve:

a. (4 points) $f(x)=\frac{5x^2-3x+11}{x^3-6x^2-7x}$

b. (5 points) $g(x)=\frac{\sqrt{3x^2+2}}{2x-1}$