Prince Sultan University
Department of Mathematics and Sciences
Math101
Major II

Fall Semester 091
Saturday, December 26, 2009
Time Allowed: 90 minutes

Student Name:	Teacher's Name:
Student ID #:	Section #:

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may **NOT borrow** a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be **NO talking** during the examination.
5. Your exam will be taken **immediately** if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. Provide an organized complete solution for each Question.
8. This examination has 11 problems. Make sure your paper has all these problems.

Problems	Max. points	Student's Points
1,2	20	
3,4	12	
5	12	
6	12	
7,8	21	
9,10,11	21	
Total	100	

Q1. (13 points) For the following system of linear equations $\begin{cases} x \geq 2 \\ 2y - x \geq 0 \\ y - 2x \geq -6 \\ x \geq 0, \quad y \geq 0 \end{cases}$.

a) Graph the solution set of the system.
b) Is the solution set bounded or unbounded?
c) List the corner points of the solution.
d) Use the information above to find the maximum and minimum values of the objective function $Z = 3x + 4y$.

Q2. (7 points) **Write down the Linear Programming Problem (Don't solve it).**

A company makes two products, one deluxe and one regular. There are 8 hours available daily on the assembly line, and 12 hours available at the painting station. Each deluxe item takes 2 hours of assembly and 2 hours of painting. Each regular item takes 1 hour of assembly and 3 hours of painting. If the profit from each deluxe item is \$420 and the profit from each regular item is \$360, how many of each should be made daily to maximize the profit?

Q3. (8 points) Find the solution for the following Linear Programming problems that have the corresponding final tableau (**Write the values of basic variables**):

a)

$$
\begin{array}{cccccccc}
P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & \text{RHS}
\end{array}
$$

$$
\left[\begin{array}{ccccccc|c}
0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 1 & \frac{-2}{3} & \frac{2}{3} \\
0 & \frac{1}{8} & 1 & 0 & \frac{-1}{8} & 0 & \frac{1}{4} & \frac{5}{4} \\
0 & \frac{19}{24} & 0 & 1 & \frac{5}{24} & 0 & \frac{-1}{12} & 55 \\
\hline
1 & \frac{19}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{3}{2} & 75
\end{array}\right]
$$

b)

$$
\begin{array}{ccccccccc}
P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & \text{RHS}
\end{array}
$$

$$
\left[\begin{array}{cccccccc|c}
0 & 0 & 0 & 1 & \frac{-3}{5} & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \frac{-6}{5} & \frac{1}{5} & \frac{3}{5} & \frac{2}{5} & 75 \\
0 & 0 & 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{4}{5} & \frac{19}{5} \\
0 & 0 & 0 & 0 & 1 & \frac{33}{5} & \frac{-4}{5} & 6 & \frac{24}{5} \\
\hline
1 & 0 & 0 & 0 & \frac{33}{5} & \frac{33}{5} & \frac{4}{5} & \frac{9}{15} & -533
\end{array}\right]
$$

c)

$$
\begin{array}{ccccccccc}
P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & s_4 & \text{RHS}
\end{array}
$$

$$
\left[\begin{array}{cccccccc|c}
0 & 0 & 0 & 0 & 1 & \frac{1}{10} & \frac{1}{10} & \frac{-1}{10} & \frac{41}{10} \\
0 & 0 & 0 & 1 & 0 & \frac{1}{20} & \frac{19}{20} & \frac{-11}{20} & \frac{79}{20} \\
0 & 1 & 0 & 0 & 0 & \frac{-1}{5} & \frac{-6}{5} & 0 & \frac{4}{5} \\
0 & 0 & 1 & 0 & 0 & \frac{3}{20} & \frac{3}{20} & \frac{-7}{20} & \frac{23}{20} \\
\hline
1 & 0 & 0 & 0 & 0 & \frac{7}{10} & \frac{57}{10} & \frac{-53}{10} & \frac{337}{10}
\end{array}\right]
$$

Q4. (4 points) Locate the Pivot number in the following tableau (**Don't solve**):

a)

$$
\left[\begin{array}{cccccccc|c}
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 10 \\
0 & -1 & -1 & -1 & 0 & 1 & 0 & 0 & -10 \\
0 & 2 & 1 & 3 & 0 & 0 & 1 & 0 & 19 \\
0 & 0 & -2 & -3 & 0 & 0 & 0 & 1 & -21 \\
\hline
1 & 5 & 7 & 6 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

b)

$$
\left[\begin{array}{cccccc|c}
0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 \\
0 & 0 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} & 1 & 2 \\
\hline
1 & 0 & -1 & -2 & -3 & 0 & 6
\end{array}\right]
$$

Q5. (14 points) Consider the following Linear Programming problem: Minimize $C = 5x_1 + 4x_2 + 2x_3$ subject to the constraints $\begin{cases} x_1 + x_2 + x_3 \geq 100 \\ 2x_1 + x_2 \geq 50 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$.

a) Write the dual maximum problem for the minimum problem.

b) Introduce slack variables and construct the initial simplex tableau.

c) Use simplex method to solve the problem.

Q6. (12 points) Use simplex method with mixed constraints to solve the Linear Programming problem.

Maximize $P = 3x_1 + 2x_2$ subject to the constraints $\begin{cases} x_1 + x_2 \le 8 \\ x_1 + 2x_2 \ge 6 \\ x_1 + 2x_2 = 6 \\ x_1 \ge 0 \\ x_2 \ge 0 \end{cases}$

Q7. (7 points) SR12000 is invested at 9% compounded semiannually.

a) What is the amount after 3 years?

b) How much interest is earned?

Q8. a) (7 points) How long will it take for an investment of \$30000 to double in value if it earns 5% compounded continuously?

b) (7 points) What rate of interest compounded quarterly will yield an effective interest rate of 7%?

Q9. (8 points) You need to borrow SR50,000 right now, but can repay the loan in 9 months. You are offered two types of loan. A discounted loan at 8% per annum and a simple interest loan at 8.5% per annum. Which type of loan should you take so that you pay as little interest as possible?

Q10. (6 points) Find the amount of annuity twelve monthly deposits of $400 at 12% compounded monthly.

Q.11 (7 points) A company establishes a sinking fund to provide for the payment of a SR100000 dept maturing in 4 years. Contributions to the fund are to be made each year. Find the amount of each annual deposit if interest is 8% per annum.