



Prince Sultan University  
Orientation Mathematics Program

MATH 002

Midterm Examination

Semester II, Term 092

Monday, April 12, 2010

Time Allowed: 120 minutes (2 hours)

Student Name: \_\_\_\_\_

Student ID #: \_\_\_\_\_

Section #: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

**Important Instructions:**

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 15 problems, some with several parts. Make sure your paper has all these problems.

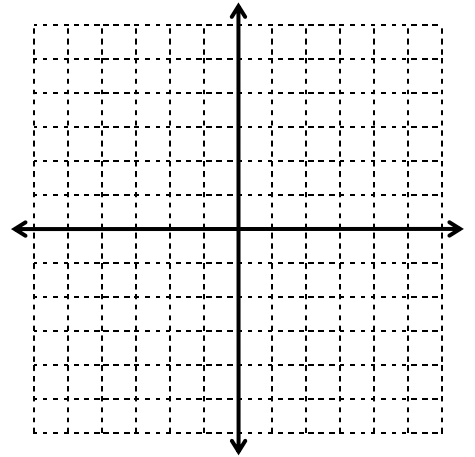
| Problems   | Max points | Student's Points |
|------------|------------|------------------|
| 1          | 30         |                  |
| 2,3,4      | 16         |                  |
| 5,6,7,8    | 17         |                  |
| 9,10,11,12 | 19         |                  |
| 13,14,15   | 18         |                  |
| Total      | 100        |                  |

Q.1 (30 points) Write only the final answer for each part

| #  | Questions                                                                                                           | Answers |
|----|---------------------------------------------------------------------------------------------------------------------|---------|
| 1  | Evaluate $\log_3(\log_b b)$                                                                                         |         |
| 2  | Evaluate $\log_{12} \sqrt{12} + \log_5 \frac{1}{125}$                                                               |         |
| 3  | Change the logarithmic expression: $\log_b a = c$ to exponential form                                               |         |
| 4  | If $\log_b a = 2$ and $\log_b c = 5$ , find the value of $\log_c a$                                                 |         |
| 5  | Condense the logarithmic expression<br>$\log x + \log(x^2 - 1) - 2 \log 7 + \log(x + 1)$                            |         |
| 6  | Find the domain of the function $f(x) = \ln(7 - x)$                                                                 |         |
| 7  | Solve $8^{x+7} = 4$                                                                                                 |         |
| 8  | Find the angle that is coterminal to $-765^\circ$                                                                   |         |
| 9  | Convert the angle $18^\circ$ to radians                                                                             |         |
| 10 | Find the measure, in degrees, of the angle $\theta$ <div data-bbox="842 1294 992 1473" data-label="Diagram"> </div> |         |
| 11 | Find the exact value of $\frac{1}{2} \sin \theta \csc \theta$                                                       |         |
| 12 | Let $\sin \theta < 0$ and $\cos \theta < 0$ . Name the quadrant in which $\theta$ lies.                             |         |
| 13 | Determine the amplitude and period of $y = -5 \cos\left(\frac{1}{2} \pi x + 4\pi\right)$                            |         |
| 14 | Find the exact value of the expression: $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$                     |         |
| 15 | Find all solutions of the equation: $\cos x = -1$                                                                   |         |

**Show all steps for each question**

Q.2 (8 points) Consider the functions  $f(x) = \left(\frac{1}{4}\right)^x$  and  $g(x) = \log_{\frac{1}{4}} x$ . By constructing tables of coordinates, graph  $f$  and  $g$  in the same rectangular coordinate system. Determine on the graph the horizontal and vertical asymptotes of  $f$  and  $g$ .



Q.3 (5 points) The formula  $P(t) = 36.1e^{0.0126t}$  models the population  $P$  of Ohio in millions, where  $t$  is the number of years after 2005.

(a) What was the population of Ohio in 2005?

(b) When will the population of Ohio reach 40 million?

Q.4 (3 points) At a certain time of day, the angle of elevation of the sun is  $35^\circ$ . Find, to the nearest foot, the height of a building whose shadow is 30 feet long.

Q.5 (3 points) Expand the logarithmic expression  $\log \left[ \frac{x^3 \sqrt[5]{x^2+1}}{(x+1)^4} \right]$  as much as possible.

Q.6 (4 points) Solve the logarithmic equation:  $\log_5 x + \log_5 (4x-1) = 1$

Q.7 (4 points) Let  $P = (4, -3)$  be a point on the terminal side of an angle  $\theta$ . Find the exact value of each of the six trigonometric functions of  $\theta$ .

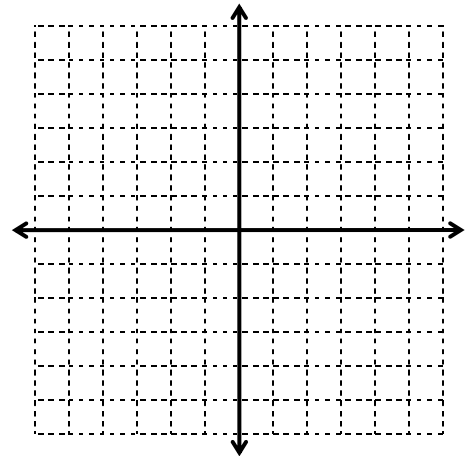
Q.8 (6 points) Find the exact value of each part. (**Show all details**)

(a)  $\cos \frac{5\pi}{3}$

(b)  $\sin 195^\circ$

Q.9 (3 points) The hour hand of a clock is 6 inches long and moves from 1 to 4 o'clock. How far does the tip of the hour hand move? Express your answer in terms of  $\pi$  and then round to two decimal places.

Q.10 (6 points) Graph one period of the function  $y = 2\sin(2x - \pi) + 1$



Q.11 (4 points) Use a sketch to find the exact value of  $\cos\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$

Q.12 (6 points) Let  $\sin \alpha = \frac{3}{5}$ ,  $\alpha$  lies in quadrant 2, and  $\sin \beta = -\frac{5}{13}$ ,  $\beta$  lies in quadrant 3. Find the exact value of  $\cos(\alpha + \beta)$ .

Q.13 (8 points) verify each identity

(a)  $\tan x (\cot x + \cos x \sin x) + \cos^2 x = 2$

(b)  $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$

Q.14 (5 points) Solve the equation  $\sin 4x = \frac{\sqrt{3}}{2}$  on the interval  $[0, 2\pi)$ .

Q.15 (5 points) Solve the equation  $\sin x \tan x = \sin x$  on the interval  $[0, 2\pi)$ .