



Prince Sultan University
Orientation Mathematics Program

MATH 002

Final Examination

Semester II, Term 072

Tuesday, June 10, 2008

Time Allowed: 150 minutes

Student Name: _____

Student ID #: _____

Section #: _____

Teacher's Name: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 18 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3	14	
4,5,6,7	19	
8,9,10	15	
11,12	16	
13,14	10	
15	11	
16	8	
17,18	7	
Total	100	

Q1. (3 points) Use your Calculator to find the value to the nearest three decimal places of each function:

a) $\log_{6.2}(235.2)$

b) $\sin(23.5)$

c) $\sec(5.7^\circ)$

Q2. (5 points) Find the **domain**, the **range**, and then **graph** the function $f(x) = \log(x + 3)$

Q3. (6 points) Solve the following equations correct to two decimal places.

a) $2^{3x+1} - 5 = 22$

b) $\log(3x - 3) = \log(x + 1) + \log(4)$

Q4. (3 points) Determine the **amplitude**, the **period**, and **phase-shift** of the function:

$$f(x) = -\pi \cos(3x - \pi) + 4 \quad \text{(Do NOT graph)}$$

Q5. (5 points) Find **the exact** value of the following expressions: (Show all your steps)

a) $\cos 50^\circ \cos 20^\circ + \sin 50^\circ \sin 20^\circ$

b) $\sin 15^\circ$

Q6. (5 points) Use a right triangle to write: $\tan\left(\cos^{-1}\left(\frac{\sqrt{4x^2 - 5}}{2x}\right)\right)$ as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

Q7. (6 points) Graph the solution set of the following system of inequalities

$$x + y \geq 3$$

$$x - y < 2$$

Q8. (4 points) Find **the vertex, focus** and **directrix** of the parabola: $(x-1)^2 = 20(y-2)$
(Do NOT graph)

Q9. (5 points) **Graph** the following equation, and give the location of the foci.
 $9(x-1)^2 + 4(y+3)^2 = 36$

Q10. (6 points) (i) **Find** the standard form of the equation of a hyperbola with foci at $(-4,0)$ & $(4,0)$ and vertices at $(-3,0)$ & $(3,0)$.

(ii) **Graph** the hyperbola.

(iii) **Find** the equations of the asymptotes.

Q11. (8 points) Verify the following identities:

a) $\frac{\sin(\alpha + \beta)}{\sin \alpha \sin \beta} = \cot \alpha + \cot \beta$

b) $\frac{\tan^2 x}{\sec x} = \sec x - \cos x.$

Q12. (8 points) Solve each trigonometric equation on the interval $[0, 2\pi)$.

a) $\cos x - 2 \cos x \sin x = 0$

b) $\sin^2 x + \cos x + 1 = 0$

Q13. (3 points) Determine whether the matrix $B = \begin{bmatrix} -2 & 1 & -1 \\ -5 & 0 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ is the multiplicative

inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$. (Show all your steps)

Q14. (7 points) (i) Use Gaussian elimination to **solve** the following system of equations. Write the solution set, if any.

$$x + y - z = 2$$

$$2x - y + 3z = 3$$

$$x - 2y + 4z = 1$$

(ii) Is the system dependent, independent, or inconsistent?

Q15. (11 points) Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

a) Find the multiplicative inverse of A

b) Find $-2A + 3I_3 B$

c) Find the matrix X that satisfies the equation: $AX - 2B = 0$

Q16. (8 points) Use Cramer's rule to solve the system:

$$x + y - z = 1$$

$$2x - y + 2z = 2$$

$$-3x + z = -1$$

Q17. (4 points) It is estimated that in t years from 2000, the population $P(t)$ of Riyadh

will be $P(t) = \frac{32}{5 + 3e^{-0.1t}}$ million.

a) What is the population of Riyadh in 2004, correct to two decimal places?

b) When does the population of Riyadh reach 6 million?

Q18. (3 points) Evaluate the determinant

$$\begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 15 & 0 & 3 \\ 4 & -9 & 2 & 1 \\ 8 & -7 & 4 & 2 \end{vmatrix}$$