Prince Sultan University
Orientation Mathematics Program
MATH 002
Final Examination
Semester II, Term 062
Saturday, June 9, 2007
Time Allowed: 150 minutes

Student Name: _____

Student ID #: _____ Section #: _____

Teacher's Name: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 18 problems, some with several parts.. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3	13	
4,5,6	13	
7	12	
8,9	16	
10,11,12	13	
13,14	9	
15,16	9	
17	8	
18	7	
Total	100	

1. (4 points) Use properties of logarithms to write the logarithmic expression $\frac{1}{3}(\log_2 x^6 - \log_2 y^9) - 5\log_2(x+1)$ as a single logarithm whose coefficient is one.

2. (3 points) Find the domain of $\log\left(\frac{x+1}{x-5}\right)$

3. (6 points) Solve the following expressions correct to two decimal places.

   (i) $6^{\frac{x-3}{4}} = \sqrt{6}$

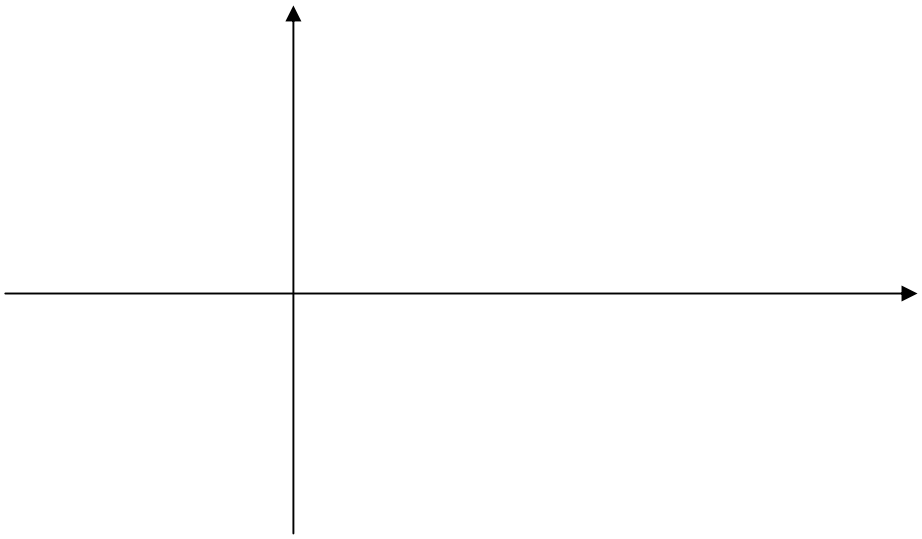
   (ii) $e^{5x-3} - 2 = 10476$

   (iii) $\log_4(3x+2) = 3$

4. (4 points) Let $\csc\theta = \frac{5}{3}$ where θ an acute angle. Find the exact value of $\cos(\theta + \frac{\pi}{2})$

5. (4 points) Use a right triangle to write $\sec(\sin^{-1}\frac{\sqrt{x^2-16}}{x})$ as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

6. (5 points) Determine the amplitude, period, and phase shift of the function $y = \frac{1}{2}\cos(3x + \frac{\pi}{2})$. Then graph one period of the function.



7. (12 points) Verify the following identities:

(i) $\dfrac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

(ii) $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x} = 0$

(iii) $\dfrac{\sin(\alpha - \beta)}{\cos\alpha \cos\beta} = \tan\alpha - \tan\beta$

8. (12 points) Solve the following equations on the interval $[0, 2\pi)$.

(i) $\sin^2 x + \sin x - 2 = 0$

(ii) $\sin x - \cos x = 1$

(iii) $7\sin^2 x - 1 = 0$

9. (4 points) Find the vertex, focus and directrix of the parabola $(y+3)^2 = 12(x+1)$

10. (4 points) Convert $9x^2 + 25y^2 - 36x + 50y - 164 = 0$ to the standard form. Then find the foci and the length of the minor axis.

11. (4 points) Graph the solution set of $\begin{cases} x - y \le 2 \\ x > -2 \\ y \le 3 \end{cases}$

12. (5 points) Find the foci and the asymptotes of $(x-3)^2 - 4(y+3)^2 = 4$. Then graph the equation.

$$2x + y = 2$$

13.(4 points) Solve the system $x + y - z = 4$. (Show all your steps).

$$3x + 2y + z = 0$$

14. (5 points) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & -12 \\ -12 & 24 \end{bmatrix}$. Perform the indicated matrix operation. If an operation is not defined, state the reason.

i. AB

ii. $2BA - \frac{1}{6}C$

15. (4 points) Evaluate the determinant $\begin{vmatrix} 0 & 4 & 0 & 2 \\ -1 & 3 & 5 & 1 \\ 2 & -2 & 0 & 7 \\ 3 & 0 & 0 & 1 \end{vmatrix}$

16. (5 points) Find the Augmented matrix and use Gaussian elimination to find the solution set of the system

$$\begin{aligned} 2x - y - z &= 4 \\ x + y - 5z &= -4 \\ 3x - 6z &= 4 \end{aligned}$$

17.(8 points) Given that $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 6 \end{bmatrix}$.

i. Find the inverse of A

ii. Use the inverse to solve the system $\begin{aligned} x + z &= 6 \\ x + y + 2z &= -2 \\ 3x + 2y + 6z &= 2 \end{aligned}$.

18. (7 points) Use Cramer's rule to solve the system of linear equations:

$$\begin{aligned} 3x + 2z &= 4 \\ 5x - y &= -4 \\ 4y + 3z &= 22 \end{aligned}$$

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