Prince Sultan University
Orientation Mathematics Program
MATH 002
Final Examination
Semester I, Term 061
Wednesday, January 24, 2007
Net Time Allowed: 150 minutes

Student Name: _____

Student ID #: _____ Section #: _____

Teacher's Name: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 20 problems, some with several parts.. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3,4	14	
5,6	13	
7,8	14	
9,10,11	14	
12,13,14	15	
15,16	10	
17,18	10	
19,20	10	
Total	100	

1. (4 points) Use properties of logarithms to expand $\log_2 \sqrt[5]{\frac{xy^4}{16}}$ as much as possible.
   Where possible, evaluate logarithmic expressions without using a calculator.

2. (4 points) Evaluate the following expressions without using a calculator
   (i) $\log_3 \frac{1}{9}$

   (ii) $\log_7 \sqrt[3]{7}$

   (iii) $2^{\log_{64} 8}$

   (iv) $(\frac{1}{49})^{\log_7 1}$

3. (3 points) The formula $S = C(1+r)^t$ models inflation, where $C =$ the value today, $r =$ the annual inflation rate, and $S =$ the inflation value t years from now. If the inflation rate is 3%, how much will a house now worth $110,000 be worth in 5 years.

4. (3 points) Use a calculator to find the value of the acute angle θ to the nearest degree.
   i. $\sin\theta = 0.2974$

   ii. $\tan\theta = 4.6252$

   iii. $\sec\theta = 1.1401$

5. (8 points) Solve the following equations to four decimal places:

(i) $\log_2(x-3) = 2 + \log_2(x+2) - \log_2 x$

(ii) $3^{2x} + 3^x - 2 = 0$

6. (5 points) Find the **exact** value of the remaining trigonometric functions of θ if

$$\tan\theta = \frac{5}{12} \quad , \quad \cos\theta < 0$$

7. (6 points) Verify the following identities:

(i) $(3\cos\theta - 4\sin\theta)^2 + (4\cos\theta + 3\sin\theta)^2 = 25$

(ii) $\dfrac{\sin(x+h)-\sin x}{h} = \cos x \dfrac{\sinh}{h} + \sin x \dfrac{\cosh - 1}{h}$

8. (8 points) Solve the following equations on the interval $[0, 2\pi)$.

(i) $3\cos^2 x = \sin^2 x$

(ii) $\cos x - 2\sin x \cos x = 0$

9. (4 points) Use a right triangle to write $\sec(\sin^{-1}\dfrac{x}{\sqrt{x^2+4}})$ as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

10. (5 points) Find the foci of $9(x-1)^2+4(y+3)^2=36$. Then graph the equation.

11. (5 points) Convert $4x^2-9y^2+8x-18y-6=0$ to the standard form. Then find the foci and write the equations of the asymptotes.

12. (4 points) Find the standard form of the equation of the parabola satisfying the following conditions : Focus: $(7,-1)$; Directrix: $y=-9$.

13. (5 points) Graph the solution set of $\begin{aligned} 4x - 5y &\geq -20 \\ x &\geq -3 \end{aligned}$

14. (6 points) Determine the amplitude, period, and phase shift of $y = \frac{-1}{2}\sin(x + \pi)$. Then graph one period of the function.

15. (4 points) Determine if $(5, -3, -2)$ is a solution of the system $\begin{aligned} x + y + z &= 0 \\ x + 2y - 3z &= 5 \\ 3x + 4y + 2z &= 1 \end{aligned}$

16. (6 points) Let $A = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 \\ -2 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Perform the indicated matrix operation. If an operation is not defined, state the reason.

i. $A(CB)$

ii. $A(2B - C)$

iii. $BA + CA$

17. (4 points) Use Gaussian elimination to find the solution set of the system

$$3x + 7y + 6z = 26$$
$$x + 2y + z = 8$$

18. (6 points) Solve the system of linear equations by using A^{-1}.

$$\begin{aligned} x - y + z &= 8 \\ 2y - z &= -7 \\ 2x + 3y &= 1 \end{aligned}$$

19. (3 points) Evaluate the determinant $\begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & -2 & 4 & 0 \\ 0 & 3 & 0 & 1 \end{vmatrix}$

20. (7 points) Use Cramer's rule to solve the system of linear equations:

$$\begin{aligned} 2x + y &= -4 \\ y - 2z &= 0 \\ 3x - 2z &= -11 \end{aligned} \quad .$$