### PRINCE SULTAN UNIVERSITY

## <u>MATH 111</u>

# **CALCULUS**

# MAJOR EXAM 1

# **26<sup>th</sup> March 2011**

Name:

<u>I.D.</u>

### **Instructors Name:**

Section:

- 1. Answer all questions
- 2. This exam consists of 1 Cover Sheet & 5 Question Sheets with 10 questions.
- 3. You can use a calculator, **NOT** a mobile phone.
- 4. No talking during the test.
- 5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1	16	
2,3,4,5,6	20	
7	18	
8	18	
9. 10	8	
TOTAL SCORE	80	
TOTAL %	100	

1) [16 points] Use the graph to find the following:

Question	Answer
a) the domain of $f$	
b) the range of $f$	
c) the zeros of $f$	
d) for what values of <i>x</i> is $f(x) \ge 0$	
e) intervals of which $f$ is increasing	
f) intervals on which $f$ is decreasing	
g) the <b>relative</b> maxima of $f$	
h) the numbers at which the maximum of $f$ occurs	
i) The value of $f(2)$	
$j)  \lim_{x \to -2^+} f(x) =$	
k) $\lim_{x \to -2^{-}} f(x) =$	
1) $\lim_{x \to -2} f(x) =$	
m) $\lim_{x \to 1} f(x) =$	

2) [4 points] Find the domain of 
$$f(x) = \frac{\sqrt{x-3}}{x^2 - 3x - 10}$$

3) [4 points] Given the function 
$$f(x) = \begin{cases} \frac{1}{2x} & x < 1\\ \sqrt{x} + 1 & 1 \le x < 3, \text{ find the value of :} \\ x^3 & x \ge 3 \end{cases}$$

- a) f(1)
- b)  $\lim_{x\to 1} f(x)$
- c) Is the function continuous at x = 1? Explain your answer
- 4) [4 points] Sketch the graph of  $f(x) = x^3$ , use this graph to sketch  $g(x) = -(x+2)^3 3$

5) [4 points] Express f(x) = -4|x-1|+2 in piecewise form without the absolute value bars.

6) [4 points] Find a general equation (in terms of x and slope m) for the family of lines that passes through the point (3, 4).

7) [18 points] Find the value of the following:

a) 
$$\lim_{x \to 3} \left( x^3 - 2x^2 + 4 \right)$$

b) 
$$\lim_{x \to 0} \frac{6x^4 - x}{-x^3 - 2x^2 + 2}$$

c) 
$$\lim_{x\to 3^+} \frac{x}{x-3}$$

d) 
$$\lim_{t \to 1} \frac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2}$$

e) 
$$\lim_{y \to 9} \frac{9 - y}{3 - \sqrt{y}}$$

f) 
$$\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$$

8) [18 points] Find the value of the following limits:

a) 
$$\lim_{x \to +\infty} \frac{(-2x+4)^4 (-x+7)}{(2x+15)^2}$$

b) 
$$\lim_{a \to -\infty} \frac{1}{a - 12}$$

c) 
$$\lim_{x \to -\infty} \frac{5x^3 - 2x^2}{x - 2x^2}$$

d) 
$$\lim_{x \to \infty} \sqrt[3]{\frac{x^2 - 2x^3}{x^3 - 2x}}$$

e) 
$$\lim_{y \to +\infty} \sqrt{y^2 + 3} - y$$

f) 
$$\lim_{x \to -\infty} \frac{2 - x}{\sqrt{3 + 6x^2}}$$

- 9) [4 points] Sketch a possible graph for a function f with the following properties:
  - i) the domain is [-1,1]ii) f(-1) = f(0) = f(1) = 0iii)  $\lim_{x \to -1^+} f(x) = \lim_{x \to 1^-} f(x) = 1$  and  $\lim_{x \to 0} f(x) = 0$

10) [4 points] Find a value for the constant k that will make the following function continuous.  $f(x) = \begin{cases} 2k^2 + kx & x \le 2\\ k^2x + 3x & x > 2 \end{cases}$