

Prince Sultan University  
Department of Mathematical Sciences

**Major III Exam**

Semester II, 2005 Spring (042)

22<sup>nd</sup> May, 2005

**MATH 111 – CALCULUS I**

**Time Allowed : 120 minutes**

**Maximum Points: 100 points**

Name of the student : \_\_\_\_\_

ID number : \_\_\_\_\_

Section : \_\_\_\_\_

**For All The Students:**

- Answer all the questions.
- This exam consists of **a total of 7 pages and 12 questions.**
- Show your working in the space provided for each question.
- Show all the key steps of your work.
- Scientific, non-programmable calculators are allowed.

Question	Maximum score	<i>Your Score</i>
Q.1	6	
Q.2	8	
Q.3	8	
Q.4	8	
Q.5	8	
Q.6	8	
Q.7	8	
Q.8	8	
Q.9	8	
Q.10	10	
Q.11	8	
Q.12	12	
<b>Total</b>	<b>100</b>	

**Q.1:** Find the local linear approximation of  $f(x) = \sqrt{1-x}$  at  $x_0 = 0$

**(6 points)**

**Q.2:** Let  $y = x^2$ . Find  $dy$  and  $\Delta y$  at  $x = 2$  with  $dx = \Delta x = 1$ .  
Then sketch  $dy$  and  $\Delta y$  on the same coordinate plane.

**(8 points)**

**Q.3:** Determine all intervals where the following function is increasing or decreasing.

**(8 points)**

$$f(x) = x^5 + \frac{5}{2}x^4 - \frac{40}{3}x^3 + 5$$

**Q.4:** If  $f(x) = ax^3 + bx^2 + cx + d$ . Determine the values of  $a, b, c$ , and  $d$  such that  $f$  has a local maximum at the point  $(0, 1)$  and a local minimum at the point  $(2, 4)$  (8 points)

**Q.5:** Locate the critical points, and classify them as stationary points or points of non differentiability. (8 points)

$$f(x) = x^{\frac{1}{3}}(x + 4)$$

**Q.6:** For the function  $f(x) = 12 + 6x^2 - x^4$ , find

**(8 points)**

- a) the intervals on which  $f$  is concave up.
- b) the intervals on which  $f$  is concave down.
- c) The x-coordinates of the inflection points.

**Q.7:** A ladder 10 m tall is leaning against a vertical wall.

**(8 points)**

The ladder is sliding down the wall at a rate of 1 m/s. At what rate is the ladder sliding away from the wall when the base of the ladder is 4m from the wall?

**Q.8:** A man starts at a point  $A$  and runs east at a rate of  $200ft / \text{min}$ . (8 points)  
One minute later, another man starts at  $A$  and runs north at a rate of  $150ft / \text{min}$ .  
At what rate is the distance between them changing 1 minute after the second man starts?

**Q.9:** Graph the following polynomial and label the coordinates (8 points)  
of the stationary points and the inflection points.

$$f(x) = x^4 + 2x^3 - 1$$

**Q.10:**

Graph the following rational function and label the coordinates of the extrema points and the inflection points.  
Show the horizontal and vertical asymptote.

**(10 points)**

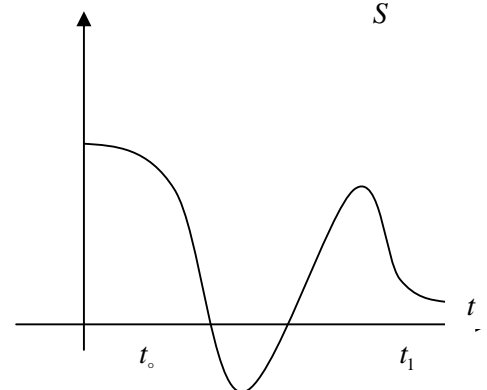
$$f(x) = x^3 + \frac{3}{x}$$

**Q.11:**

The position function of a particle moving along a horizontal x-axis is shown  
In the following figure. ( *Explain your answer* )

**(8 points)**

- a) Is the particle moving left or right at time  $t_0$  ?
- b) Is the acceleration positive or negative at time  $t_0$  ?
- c) Is the particle speeding up or slowing down at time  $t_0$  ?
- d) Is the particle speeding up or slowing down at time  $t_1$  ?



**Q.12** : The position function of a particle is moving along a coordinate line is given by  $s(t) = t^4 - 4t^3 + 10$  ,  $t \geq 0$  where  $s$  is in meters(m) and  $t$  is in seconds(s). (12 points)

- a) Find the velocity and acceleration functions
- b) Find the position, velocity, and acceleration when  $t = 2$  s
- c) At what time is the particle stopped?
- d) When is the particle speeding up? Slowing down?
- e) Analyze the motion of the particle for  $t \geq 0$  . Give a schematic picture of the motion.