``

Prince Sultan University

Department of Mathematical Sciences

Major II Exam

Semester II, 2005 Spring (042)

23rd April, 2005

MATH 111 – CALCULUS I

Time Allowed : 100 minutes
Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 7 pages and 10 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	*Your Score*
Q.1	10	
Q.2	6	
Q.3	28	
Q.4	10	
Q.5	6	
Q.6	10	
Q.7	10	
Q.8	5	
Q.9	10	
Q.10	5	
Total	**100**	

Q.1 **For** $y = 3x^2 - 2$ **(10 points)**

a) Find the average rate of change of y **with respect to** x **over the interval** $[1, 4]$**.**

b) Find the instantaneous rate of change of y **with respect to** x **at** $x_0 = 2$

Q.2 **Use the definition of the derivative to find** $f'(x)$**, where** $f(x) = x^2 - x$ **.** **(6 points)**

Q.3: **Find** $\frac{dy}{dx}$ **for the following functions:** **(simplify your answer as much as possible)**

a) $y = (x-2)^2(3-2x)^3$ **(28 points)**

b) $y = \dfrac{\sin x}{x \cos x}$

c) $y = \sqrt{\sin(7x + \ln(5x))}$

d) $y = x(\sin^{-1} x)^2$

e) $y = e^{\tan^{-1}\sqrt{3x}}$

f) $e^{x-y} = \ln\left[\dfrac{x}{y}\right]$

g) $y = x^3 \ln(\cos x^3) + \cos(\ln x^2)$

Q.4: **Determine the equation of the tangent line to the given curve at the indicated point or x-value:**

a) $y = x^2 - 2x$ **at** $x = 2$ **(10 points)**

b) $xy^3 - 3x^2 = 5$ **at** $(1, 2)$

Q.5: **Assume that** $h(x) = f(g(x))$ **, where both** f **and** g **are differentiable functions.**

If $g(-1) = 2$**,** $g'(-1) = 3$**, and** $f'(2) = -6$ **, what is the value of** $h'(-1)$ **?** **(6 points)**

Q.6 **Find the values of x and y at which the curve of the following function has horizontal tangent lines.** **(10 points)**

$$4x^2 + y^2 - 8x + 4y + 4 = 0$$

Q.7 **Use implicit differentiation to find $\frac{dy}{dx}$:** **(10 points)**

a) $\sin(x + y) = y \cos x$

b) $e^{xy} = x^2 y$

Q.8: **Use the method of logarithmic differentiation to find $\frac{dy}{dx}$:** **(5 points)**

a) $y = x^{\ln x}(\sec x)^{3x}$

b) $y = \dfrac{e^x\sqrt{x^2+3}}{(x^3+2)^5(x^2+1)^2}$

Q.9: **Find $\frac{d^2y}{dx^2}$ for the following functions: (simplify your answer as much as possible)**

(10 points)

a) $y = x\sin x - 3\cos x$

b) $y = \frac{x-1}{x+1}$

Q.10: **Find $\frac{dy}{dx}$ if $y^3t + t^3y = 1$ and $\frac{dt}{dx} = \frac{1}{\cos t}$** **(5 points)**