Prince Sultan University

Department o f Mathematical Sciences

Major I Exam

Semester I, 2005 Fall (051)

19th October, 2005

MATH 111 – CALCULUS I

Time Allowed : 100 minutes **Mr. Khaled Naseralla**

Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 7 pages and 15 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	*Your Score*
Q.1	6	
Q.2	3	
Q.3	6	
Q.4	8	
Q.5	4	
Q.6	4	
Q.7	4	
Q.8	6	
Q.9	18	
Q.10	12	
Q.11	5	
Q.12	12	
Q.13	4	
Q.14	4	
Q.15	4	
Total	**100**	

Q.1 **: Use the equation :** $y = 2 + \sqrt{x}$ **to answer the following questions:** **(6 points)**

a) For what value(s) at x **is** $y = 8$ **?**

b) For what value at x **is** $y = 0$ **?**

c) Does y **have a minimum value? A maximum value? If so, find them.**

Q.2: **If a line passes through** $(2,1)$ **with slope** $\frac{2}{3}$**, find a second point on the line and then graph it** **(3 points)**

Q.3: **(3 points each)**

a) Find an equation of a line passing through $(3,-2)$ **and perpendicular to** $x + 3y - 2 = 0$

b) State the geometric property common to all the lines in the family:

$$y = m(x-2) + 3$$

Q.4: **Find the natural domain of each of the following functions:** **(4 points each)**

a) $f(x) = \dfrac{\sqrt{8-4x}}{\sqrt{2x-1}}$

b) $\dfrac{4x}{x^2+2x-8}$

Q.5: **Use the definition of the limit to prove the following limit.** **(4 points)**

$$\lim_{x\to 2}(3x+2) = 8$$

Q.6: **Find a number** δ **such that** $|f(x)-L| < \varepsilon$ **if** $0 < |x-a| < \delta$ **, given that** $\varepsilon = 0.1$ **(4 points)**

$$\lim_{x\to 1}(5x^2+1) = 6$$

Q.7: **Express the function in piecewise without the absolute values.** **(4 points)**

$$f(x) = |4+x| - 2|x-2|$$

Q.8: **Given** $f(x) = \sqrt{x+1}$ **and** $g(x) = x^2 - 1$ **find:** **(3 points each)**

a) $(f \circ g)(x)$ **and state its domain**

b) $(g \circ f)(x)$ **and state its domain**

Q.9: **Find the limits:** **(18 points)**

a) $\lim_{y \to -2} \dfrac{(y^2 - 4)}{3y + 6}$

b) $\lim_{x \to -\infty} \dfrac{7x^2 - x + 11}{4 - x}$

c) $\lim_{x \to -\infty} \dfrac{x + 3}{\sqrt{9x^2 - 5x}}$

d) $\lim_{x \to -1} \frac{1}{x^2 + 2x + 1}$

e) $\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{4 - x}$

f) $\lim_{x \to 0^+} \frac{x(x^2 - 1)}{x^2}$

Q.10: **For the function,** $f(x) = \begin{cases} x^2 + x - 1 & -3 < x < -1 \\ x + 1 & -1 \le x < 1 \\ 3 - x^2 & 1 \le x < 2 \\ \frac{x}{2} - 2 & 2 \le x < 3 \end{cases}$ **find the following:** **(12 points)**

a) $\lim_{x \to 2^+} f(x) =$

b) $\lim_{x \to 2^-} f(x) =$

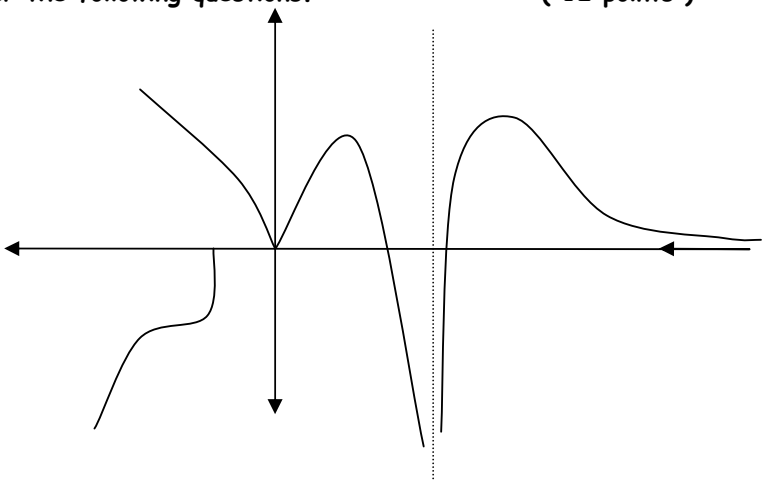
c) $\lim_{x \to 2} f(x) =$

d) Is the function continuous at $x = -1$ **(Explain why or why not)**

Q.11: **Sketch the graph of** $f(x)=\dfrac{5}{x^2-3x-4}$ **(5 points)**

Show the vertical and horizontal asymptotes

Q.12: **Use the graph to of the function to answer the following questions.** **(12 points)**



a) $\lim\limits_{x\to 2^+} f(x) =$

b) $\lim\limits_{x\to \infty} f(x) =$

c) $\lim\limits_{x\to -2} f(x) =$

d) $\lim\limits_{x\to 4} f(x) =$

e) $\lim\limits_{x\to -\infty} f(x) =$

f) Determine all the numbers, if any, at which the function is discontinuous.
(Give the reason for the discontinuity)

Q.13: **Find a value for the constant k that will make the following function continuous.** **(4 points)**

$$f(x)=\begin{cases} kx^2 & x \le 2 \\ 2x+k & x > 2 \end{cases}$$

Q.14: **Suppose that f and g are continuous functions such that** **(4 points)**

$\lim_{x\to 3}[f(x)+4g(x)]=13$, **and** $f(3)=1$.

Find $g(3)$

Q.15: **Find an equation for the graph that has the form:** **(4 points)**

$$y = D + A\cos Bx$$