# Prince Sultan University

## Department of Mathematical Sciences

**Semester II, 2014 SPRING (Term 132)**
**April 21, 2014**

# MATH 111 – Calculus I
# Major II Exam

**Time Allowed : 60 minutes**
**Maximum Points : 60 points**

Name of the student: _____

ID number : _____

Section : _____

***Important Instructions:***

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 8 problems, some with several parts and a total of 5 pages. Make sure your paper has all these problems.

| Question | Maximum score | Your Score |
|---|---|---|
| Q.1 , Q.2 , Q.3 | 32 | |
| Q.4 , Q.5 , Q.6 | 16 | |
| Q.7 , Q.8 | 12 | |
| | | |
| Total | 60 | |

**Show Your Steps**

**Q.1 ( points):** Use the limit definition of derivative to find the derivative of $f(x)=\frac{1}{\sqrt{x-3}}$

**Q.1 (24 points):** Find the derivative **(Simplify as much as possible)**

(i) $f(x)=\sqrt{x}+\frac{2}{\sqrt[3]{x^4}}-5x^8$

(ii) $y=\frac{e^{5x}\sqrt{x^3-4}}{(x^2+4)^4(x^3-1)^3}$ or $y=\sqrt[5]{\frac{(2x^2+5)^3\tan^6(x)}{\sqrt{x+1}}}$

(iii) $y=3^{2x^2}.\sqrt{x}$

(iiii) $y=\ln\left[\frac{(x-2)^3}{\sqrt{2x-1}}\right]$

(iv) $y=(x^2)^{4x}$ or $y=x^{\sec(x)}$

(v) $f(x)=\left(1+\cos^3(x^4)\right)^{10}$

(vi) $f(x)=\sin\sqrt{\ln(1-3x)}$

(vii) $y=\sqrt{x}\ \sin^{-1}\left(\sqrt{x}\right)$ , $x\in(0,1)$

(iv) $f(x)=\sqrt{\cos^{-1}(x^2)}$

(v) $y=\sin^{-1}(\sqrt{x+1})$ , $x\in(-1,0)$

(viii) $y=e^{3x}\ln\left(\tan^{-1}(6x)\right)$

(v) $y=\tan^3\left(\sqrt{\cot(7x)}\right)$

(vi) $f(x)=e^{xy}-x^3+3y^2=11$ or $x^3y^2-5x^2y+y=\sin(x)$

**Q.2 (6 points):** Find $\frac{d^{82}}{dx^{82}}\left(x\cos(x)\right)$

**Q.2 (6 points):** Let $f$ and $g$ be functions such that $f(1)=-1$ , $f'(1)=2$ , and $g'(-1)=-4$.
Let $F(x)=\left(f(x)\right)^2-(g\circ f)(x)$. Find $F'(1)$

**Q.2 (6 points):** Find all the points $(x,y)$ on the curve of $y=x^3+2x^2-x+2$ where the tangent line has slope $-2$

**Q.2 (6 points):** At what point(s) is(are) the tangent line to the graph of $y=2x^3-8x+1$ is perpendicular to the line $2y-x+1=0$.

**Q.2 (6 points):** Find the limit:

i) $\lim_{x\to 0}\frac{5x^3-3\sin^2(5x)}{x^2}$

ii)

**Q.2 (6 points):** Find an equation of the tangent lines to the graph of $x^2+2xy-y^2+x=2$ at $x=1$

**Q.3 (4 points)**:

Q.4 (5 points):

**Q.5 (5 points):**

**Q.6 (6 points):** Two cars start moving from the same point. One travels north at $60$ km/h and the other travels west at $80$ km/h. At what rate is the distance between the cars increasing $2$ hours later?

**Q.6 (6 points):** A spherical snowball is melting such that its volume is decreasing at a rate of $0.5\ \text{cm}^3/\text{min}$. When the diameter is $8$ cm, at what rate is the radius decreasing?

**Q.7 (6 points):** Find all numbers **c** that satisfy the conclusion of the Mean Value Theorem of the function $f$ on the interval $[-1,2]$, where $f(x)=x^3-2x$.

**Q.8 (6 points)**: Find the **absolute** minimum and maximum values of $f$ on the given interval.

$f(x) = 3x^4 - 4x^3 - 12x^2 + 1 \quad ; [-2, 3]$